

# Rock mounted iconic lighthouses under extreme wave impacts: Limit Analysis and Discrete Element Method

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## Abstract

This paper deals with the resilience of rock mounted lighthouses impacted by extreme waves. The investigated lighthouse of Wolf Rock is built with large and meticulously dovetailed granite blocks. For this structural typology, uplift of blocks and separation of joints is possible under intense wave impacts. Sliding between subsequent courses of stones is limited by the existence of vertical keys between the blocks. The lateral forces that can trigger uplift and rocking, a highly nonlinear behaviour, are calculated with the use of Limit Analysis method through an iterative procedure written in Python programming language. Three different overturning mechanisms, which correspond to different hypotheses about the working section of the lighthouse, are considered for the uplift. The most conservative limits, i.e. with only a portion of the circular section taking the load, are representative for small and medium waves which can cause a small uplift and very local opening of a joint. The mechanism that considers the whole section, i.e. less conservative, is more representative for very strong wave impacts. The sliding failure mechanism is also considered. The Limit Analysis thresholds are validated with Discrete Element (DE) time-history analyses using the commercial software 3DEC. Intense uplift and rocking take place, and are especially present when the less conservative Limit Analysis thresholds are exceeded. The study also revealed the beneficial role of the vertical keys, without which the lighthouse would fail due to intense sliding before the overturning mechanisms are activated.

**Keywords:** Limit analysis, Discrete Element Method, historic lighthouse, rocking, wave impact

## Introduction

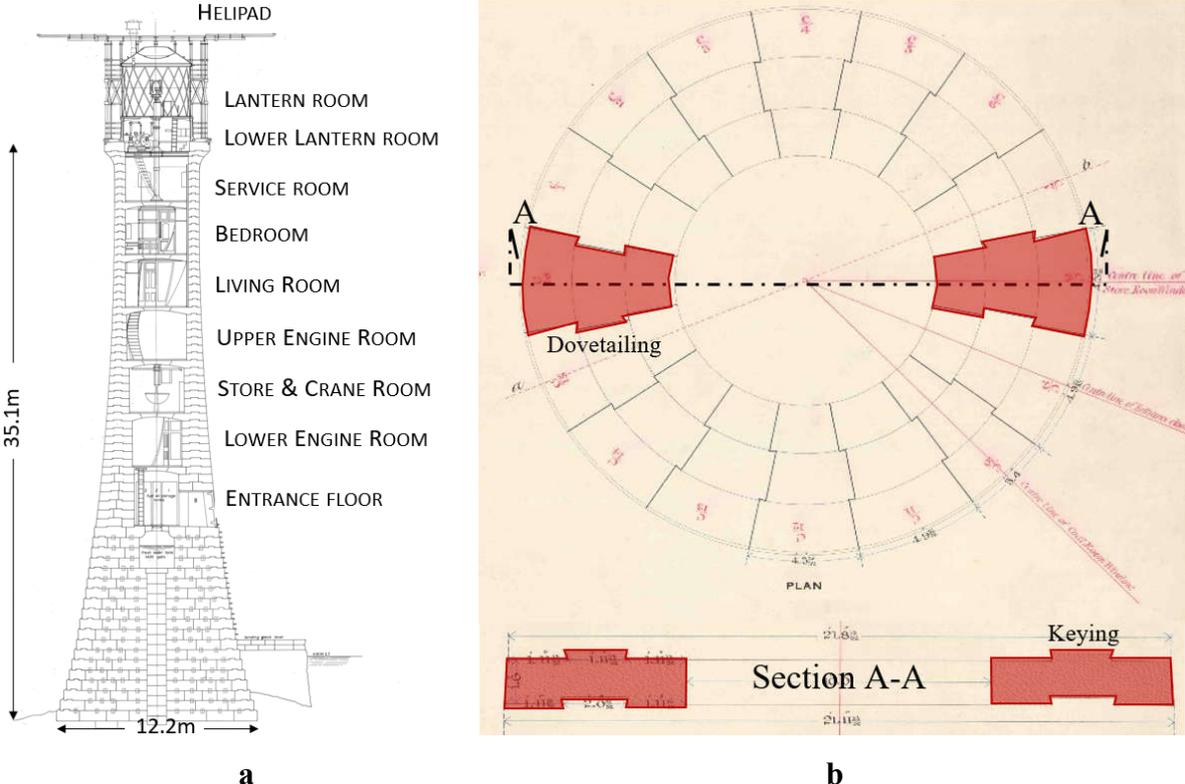
Lighthouses on hostile and exposed rocks around the British Isles and Ireland have been resisting the impacts of extreme waves for over a century. However, the history of these landmarks of engineering has not been smooth. Lighthouse engineering has evolved after repeating collapses of under-designed structures and the subsequent upgraded design. The majority of the surviving rock-mounted lighthouses in the area are built based on an ingenious design: a tapered masonry structure with large-scale interconnected blocks, proposed by John Smeaton who designed a lighthouse of Eddystone in the mid-18th century. Prior to this design, three other lighthouses on the same rock had failed. The first presented unrepairable damages in its first winter, the second collapsed after a winter storm in its fourth year, and the third caught fire nearly 50 years after its construction. Though more resilient than their predecessors, plenty of the existing rock-mounted lighthouses have manifested uplift and motion after intense wave impacts. The

importance of the lighthouse network to the safety of navigation, in combination with the heritage value of these iconic lighthouses, provided the motivation for this structural analysis. The uplift and rocking behaviour of slender structures was first introduced by Housner [1]. His work evidenced that the structural behaviour of bodies capable of uplifting differs significantly to the one of continuous structures. Later studies verified the complexity of the rocking behaviour [2]–[4]. However, all of these studies are focused on base excitation and not on lateral wave impacts. Although plenty of research has been devoted on the estimation of wave impacts [5]–[7], little has been done regarding wave impacted rock lighthouses [8].

This paper presents the application of the limit analysis method on a masonry lighthouse. At first, the limits of wave impact intensity for each impact height are calculated. The activated mechanisms, i.e. overturning or sliding, are calculated for Wolf Rock lighthouse. Then, the limit analysis results are confronted with the time-history numerical results of a Discrete Element Method (DEM) model for wave impacts calculated for the specific location and structure.

### Wolf Rock lighthouse

The rock mounted lighthouse of Wolf Rock is located around 13 km south-west from the point of Land’s End, in the menacing coastline of Cornwall, UK. The construction of the current lighthouse which survives till today was initiated in 1862 and finished in 1869. Prior to this structure, the Wolf Rock had seen a series of four beacons failing after violent storms and plenty of designs for new lighthouses being abandoned as insufficient to resist the colossal wave impacts [9].



**Figure 1. Wolf Rock lighthouse: (a) original section drawing and (b) details of dovetailing and keying for a course of stones.**

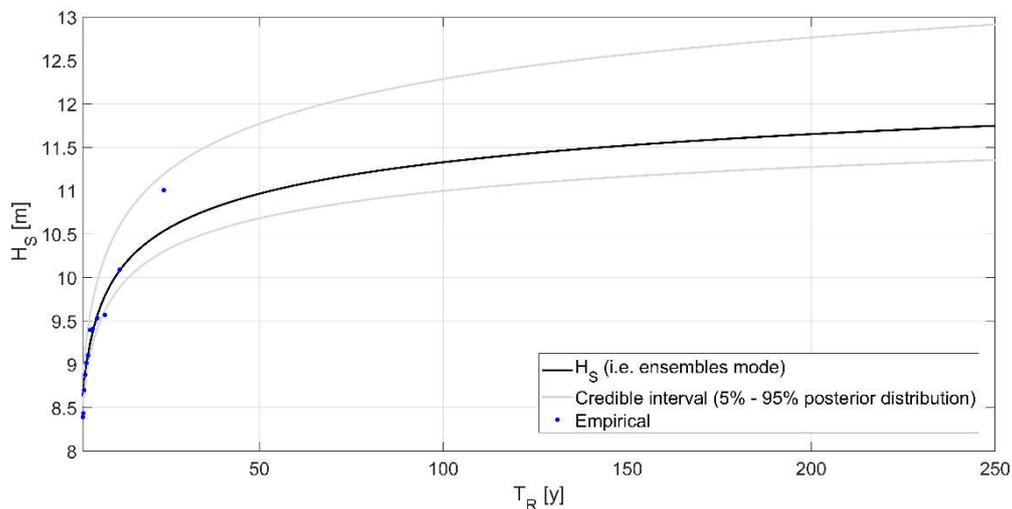
The lighthouse consists of a granite masonry body that is 35.1 m high and has a 6.1 m high lantern at the top. The diameter of the granite body is 12.2 m at the base and gradually decreases

to 5.2 m near the top. The masonry structure consists of 6 vaulted levels, plus the lantern structure on the top. The wall thickness varies between 2.37 m at the entrance level and 0.69 m at the upper level. Finally, a steel frame helideck was constructed on the top of the masonry body in the early 1970s for facilitating the movement of personnel and supplies.

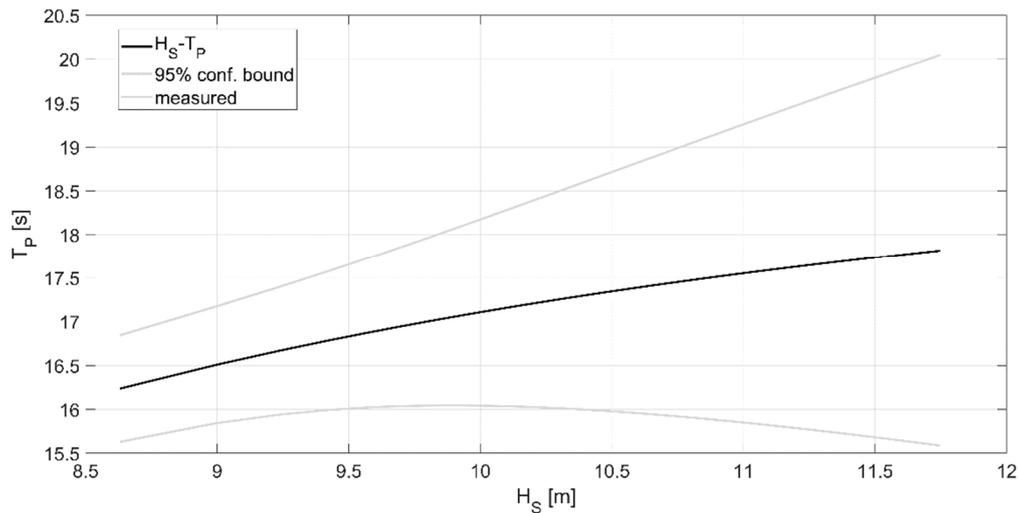
The horizontal and vertical interlocking of the granite blocks through dovetails in the vertical courses and keys in the horizontal courses is shown in Figure 1b. In this structural typology, apart from uplift, no other high relative movement between blocks is possible without fracture of the dovetailed connections. Sliding, along the horizontal joint between two successive courses of stones, is also blocked by the vertical key connections. The original drawing suggest that the height of the key is around 7.6 mm high. Although most lighthouses of this typology have the same keying technique, Wolf Rock bears additional vertical connections on the lower third of its height. This is an evidence that the designer engineer, James Nicholas Douglas, was indeed concerned about the severity of the wave impacts which could cause uplift or sliding.

### Extreme wave loading

Until now, a theoretical description of the loading condition induced by the breaking waves on emerged cylinders is not available. Thus, due to this lack and to the impellent necessity to perform a survivability assessment of these ancient structures, the method of Wienke and Oumeraci [5] is applied as final tool to describe the impulsive wave load. The method consists of five main steps: *i)* The extreme offshore wave climate is identified by means of statistical extreme analysis. The Generalised Pareto Distribution, in combination of Peak Over Threshold method, is used through Bayesian inference [10] in order to identify the extreme significant wave heights ( $H_S$ ) Figure 3, while the relationship between the significant wave height and peak period ( $T_P$ ) is described by means of exponential equation, Figure 3 [11], [12]. The final result is a set of return periods, significant wave heights and peak periods describing the offshore wave climate.

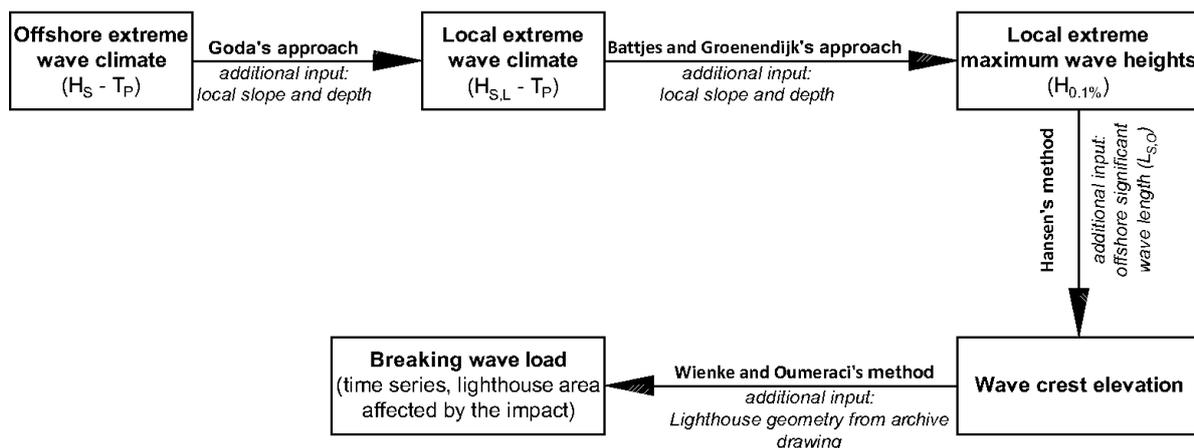


**Figure 2. Wolf Rock Lighthouse offshore significant wave height ( $H_S$ ) vs return period**



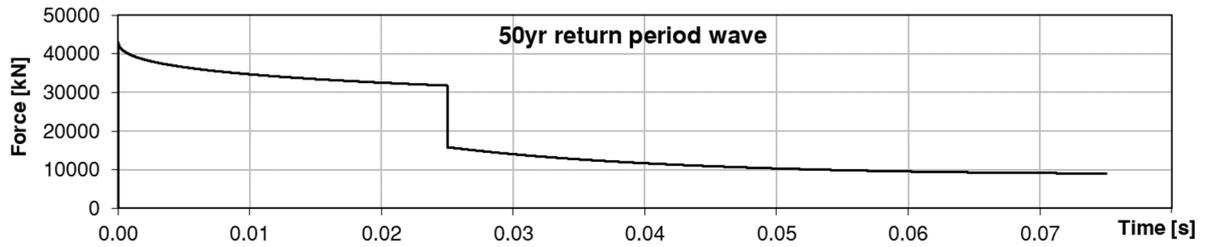
**Figure 3. Wolf Rock Lighthouse offshore peak periods ( $T_P$ ) vs significant wave height ( $H_s$ )**

ii) In the surrounding area of the Wolf Rock, the bottom topography can be represented by very steep bottom that never reach shallower water condition. The approach of Goda [13] was applied to calculate the wave transformation from offshore to the lighthouse site identifying the local significant wave height, ( $H_{s,L}$ ). iii) The effects of the restricted depth-to-height ratio and of breaking wave on the maximum wave height are considered by means of Battjes and Groenendijk's method [14], therefore, the design breaking wave height is assumed to be  $H_{0.1\%}$ . iv) Finally, the crest elevation with respect to the still water level is calculated according to Hansen's method [15]. v) Wienke and Oumeraci's method [5] is, then, applied considering the variation of the average radius of the lighthouse along to the calculated impact area. Load distribution is kept constant both in horizontal and vertical direction, while frontal area affected by the load distribution is considered included between  $\pm 30^\circ$  from the wave dominant direction. The process is summarised in Figure 4.



**Figure 4. Summary of the process leading to the breaking wave load description.**

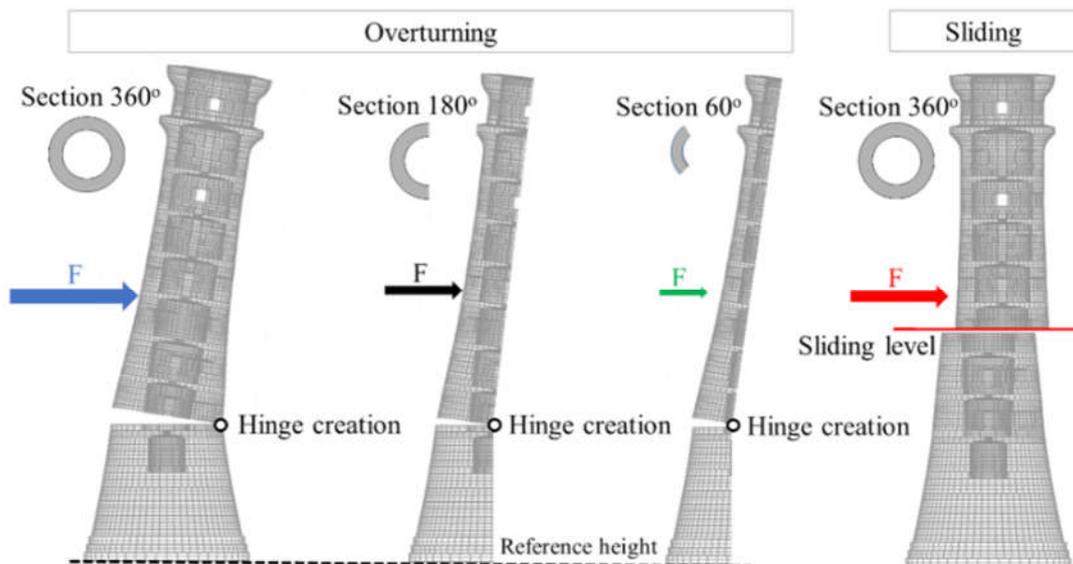
For a wave impact with return period equal to 50 years, the total impact duration is equal to 0.075 s, and the maximum impact force, at  $t = 0$ , is 43031 kN. The force is applied between the 21<sup>st</sup> and the 35<sup>th</sup> course on a frontal section of  $60^\circ$  with uniform distribution. Thus, the load is applied on 15 courses and the force resultant is at a height of 16.37 m from the base of the structure and 12.1 m from the sea level. The time-history of the total force that this impulsive wave applies on the structure is presented in Figure 5.



**Figure 5. Time-history of the impulsive wave total force applied on the structure**

### Limit analysis

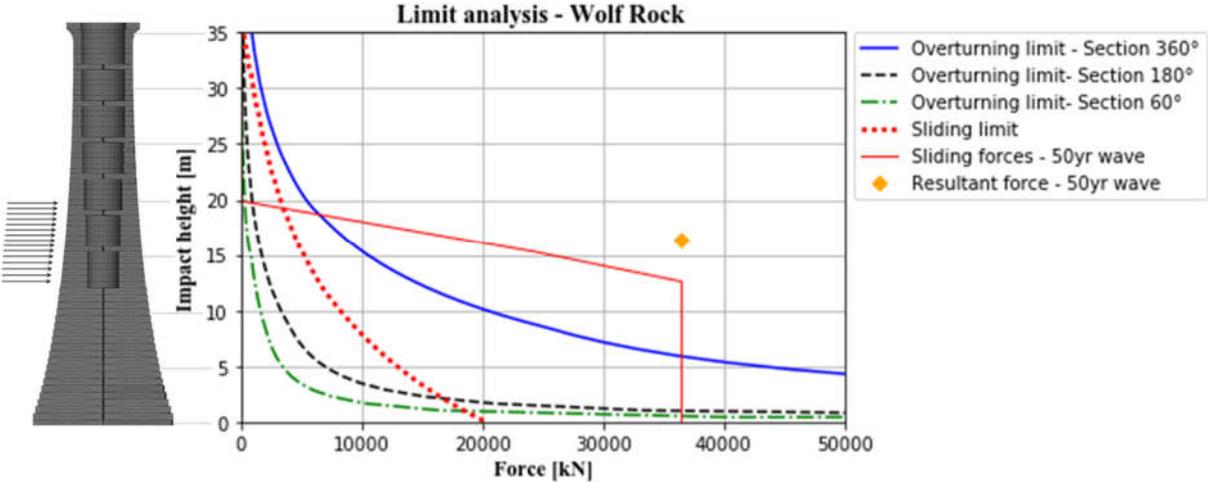
The limit analysis method calculates the magnitude of lateral force that is necessary for triggering a failure mechanism such as overturning or sliding (Figure 6). For overturning, the equilibrium of moments around a rotation hinge is calculated between the stabilisation forces, i.e. self-weight, and the external forces. For sliding, the equilibrium of horizontal forces is calculated by comparing the stabilisation forces, i.e. friction in horizontal joints, and the external forces. Regarding the overturning, three different mechanisms were considered. The first takes into account the whole section of the lighthouse (Figure 6a), the second considers only the front half section (Figure 6b), and the last mechanism considers only a frontal section of  $60^\circ$  (Figure 6c) which coincides with the impact section of the impacting wave [5]. Although the last two mechanisms ( $180^\circ$  and  $60^\circ$ ) are not realistic since the lighthouse is not fractured and therefore behaves as a continuous body, their calculation is useful for estimating the magnitude of external force that can cause a partial uplift. It has to be stated that the activation of an overturning mechanism is reversible. This means that for small duration impacts there can be some uplift and rocking but this does not necessarily mean overturning and collapse [1]–[3]. Note also that the existence of vertical keys for this lighthouse prevents any large sliding and therefore collapse due to sliding. Nevertheless, finite sliding due to small gaps between the vertical keys of the joints is still possible.



**Figure 6. Failure mechanism for limit analysis: (a) overturning for  $360^\circ$  section, (b) overturning for  $180^\circ$  section, (c) overturning for  $60^\circ$  section, (d) sliding for  $360^\circ$  section**

A drawback of the limit analysis is that the magnitude of activation force that is necessary for triggering the overturning mechanism depends on the selection of the hinge. Therefore, many hinge positions have to be tried in order to find the mechanism with the lowest activation force. Similarly, for the sliding mechanism, different joint levels have to be considered. To perform these calculations, an iterative procedure was written in Python 3.6 programming language. The self-weight of each course of stones is calculated based on the detailed geometrical data obtained during the archival research. All possible positions of horizontal activation forces are considered. Subsequently, all possible hinge or sliding levels are regarded for each external force scenario. The results for each height position of external force and the necessary magnitude for activation of the respective failure mechanism are presented in Figure 7.

The vertical axis in Figure 7 corresponds to the impact height and the horizontal axis shows the impact force that is necessary for the activation of each failure mechanism. The limit analysis curves for overturning (continuous blue, dashed black and green dash-dot green) and the sliding (dot red) are presented. The activation force decreases for increasing impact heights, which illustrates the importance of the impact height to the structural stability. The marker at force equal to 43031 kN and impact force equal to 16.37 m represents the resultant force of the 50 years return period wave which stands much higher than all limit analysis curves for overturning. This suggests that intense uplift and rocking is expected for this wave impact. In the same graph, the sliding forces, due to the wave impact, that act on each course are plotted as continuous red line. If this continuous line crosses the red dot line that represents the sliding limit, activation of sliding mechanism is possible. However, if vertical keying is present, such a mechanism cannot be activated unless the joint opens more than the height of the vertical keys or there is a rupture of the keys. The results for the 50 years wave shown in Figure 7 suggest that sliding would have been detrimental without the presence of keying.



**Figure 7. Limit analysis curves for overturning (continuous, dashed and dash-dot lines) and sliding (dotted line)**

**Numerical analysis**

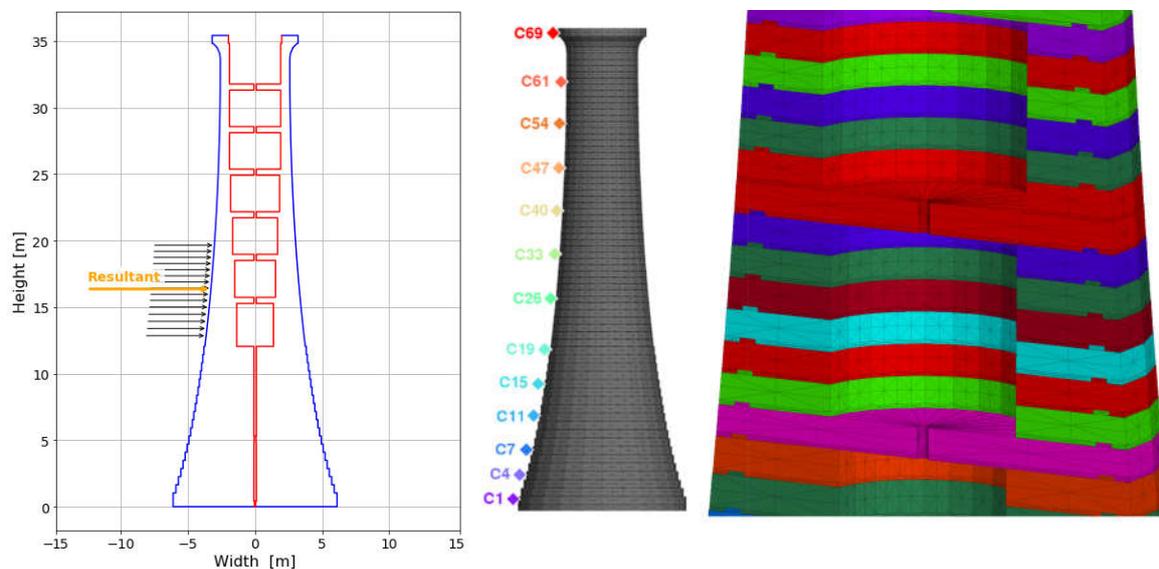
The numerical modelling of the structure was based on the Discrete Element Method (DEM) introduced by Cundall [16]. The method regards the structure as an assembly of discrete blocks and the solution is based on a time algorithm of sufficiently small time-steps. This solution scheme is described by Cundall (1976) as the DEM cycle. The cycle consists firstly on the calculation of the block motion in terms of velocity and acceleration, which are assumed to be constant within a given time-step. As the blocks move relatively, new contacts between blocks are detected and the relative contact velocities and forces are updated with the use of a force–

displacement law. Finally, the new forces for each block centroid are calculated and the new block motion is updated with the application of Newton's second law. The relatively simple theory behind the DEM circle makes the method particularly efficient for reproducing structural response of rigid bodies that is characterised by large displacements and separation between blocks [17].

The three-dimensional numerical model of the lighthouse is developed with the use of the DEM software 3DEC [18]. Each course of stones is modelled as an independent rigid block. The vertical keys were also modelled in detail, hence impeding large sliding unless significant uplift takes place. The Coulomb friction law is implemented for the joints between blocks with zero cohesion and an angle of friction equal to  $30^\circ$ . Moreover, the joint is given normal stiffness equal to  $5.93 \cdot 10^{10}$  Pa/m and shear stiffness equal to  $4.45 \cdot 10^{10}$  Pa/m. The specific weight of the masonry blocks is taken equal to  $2463 \text{ kg/m}^3$ , which corresponds to granite similar with the one used for Fastnet lighthouse [19]. Additional mass is added to the top course in order to account for the mass of the lantern and helideck that were not introduced to the model. Based on previous experimental work [19], mass proportional Rayleigh damping is adopted ( $\alpha = 1.57$ ,  $\beta = 0$ ). The time-step for each DEM circle is equal to  $3.41 \cdot 10^{-5}$ .

### Limit analysis vs DEM results

The DEM model is tested for various impact intensities by scaling the original wave time-history. The maximum intensity of the scaled waves is presented herein normalised for the force calculated by the limit analysis for the  $360^\circ$  section. For instance, the original wave calculated for 50 years return period corresponds to a normalised force of 4.94. A total of 26 analyses are carried out for normalised wave forces varying between 0.01 and 4.94.

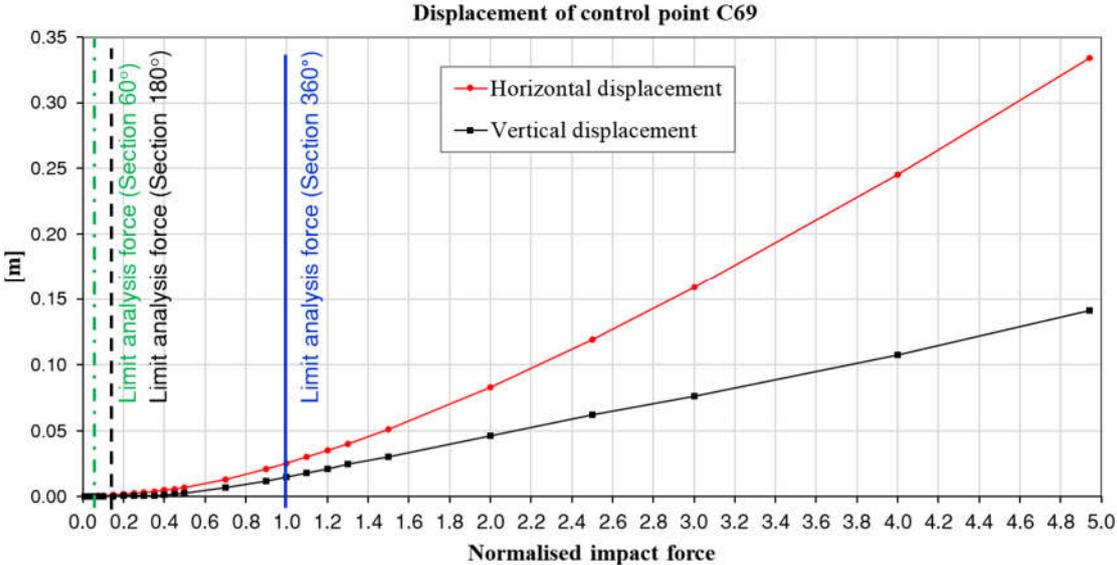


**Figure 8. (a) Wave impact on Wolf Rock; (b) DEM model and control points; (c) DEM model section**

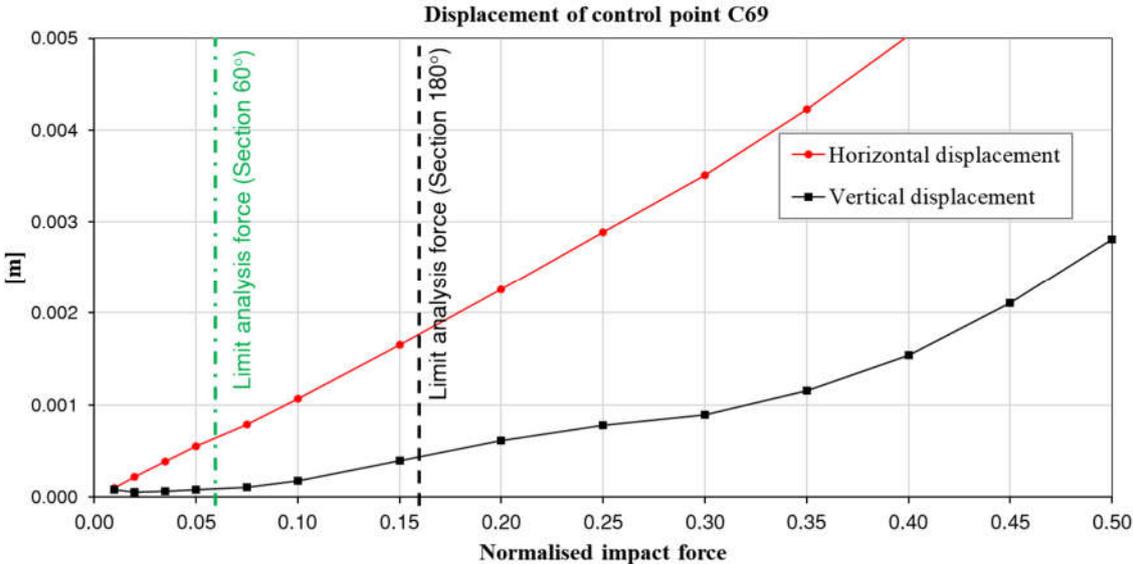
The qualitative results of the tests suggest that the DEM model is able to reproduce a realistic structural response for the wave impact. Although the courses are modelled as rigid blocks, the structure is deformable due to the joint interfaces which behave as unidirectional springs. The stiffness of the joints was calculated in order that the model behaves with an equivalent modulus of elasticity equal to 30 GPa in the vertical direction [19].

The curves presented in Figure 9 and Figure 10 testify that the correlation between the impact forces and the structural response in terms of displacements is not linear. Both the vertical and

the horizontal displacement curves begin as linear for small impact intensities but they become parabolic for stronger impacts. Note that the transition from linear to parabolic shape is gradual and thus cannot be attributed to the exceedance of a strict threshold. The transition phase is clearly presented Figure 10 for normalised forces ranging between 0.01 and 0.5. The finding that there is no specific threshold over which the nonlinearity is triggered, supports the approach of using multiple assumptions (60°, 180°, 360° section), for the limit analysis as guides for the structural assessment. The trend of the vertical and horizontal displacement curves is linear for forces near the most conservative limit analysis assumption (60° section), and becomes nonlinear much earlier than the less conservative threshold (360° section). This unclear boundary between linearity and nonlinearity can be explained by the dynamic nature of the impact force that has a duration of only 0.075 s. For impacts of much higher duration, the structural response would appear more discretised and would resemble the assumptions of the limit analysis.

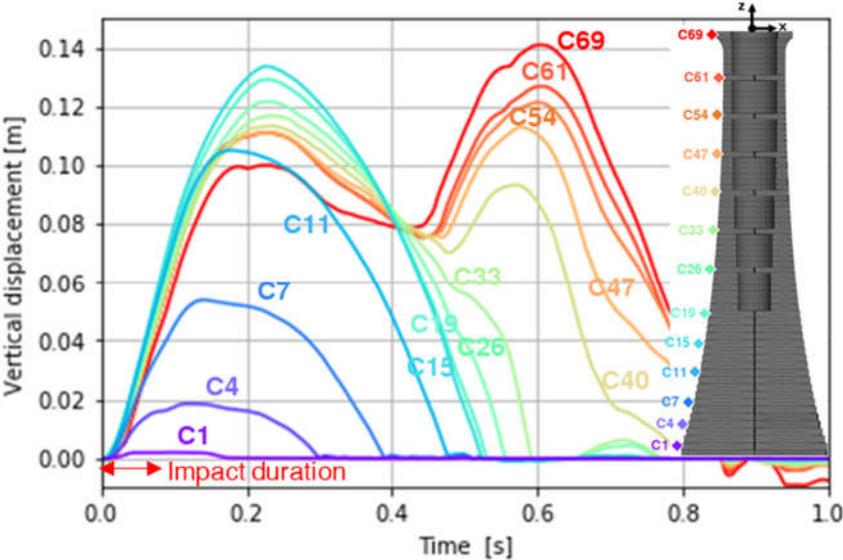


**Figure 9. Maximum horizontal and vertical displacement from DEM analyses together with limit analysis threshold values; range of normalised impact forces 0.01 - 4.94**

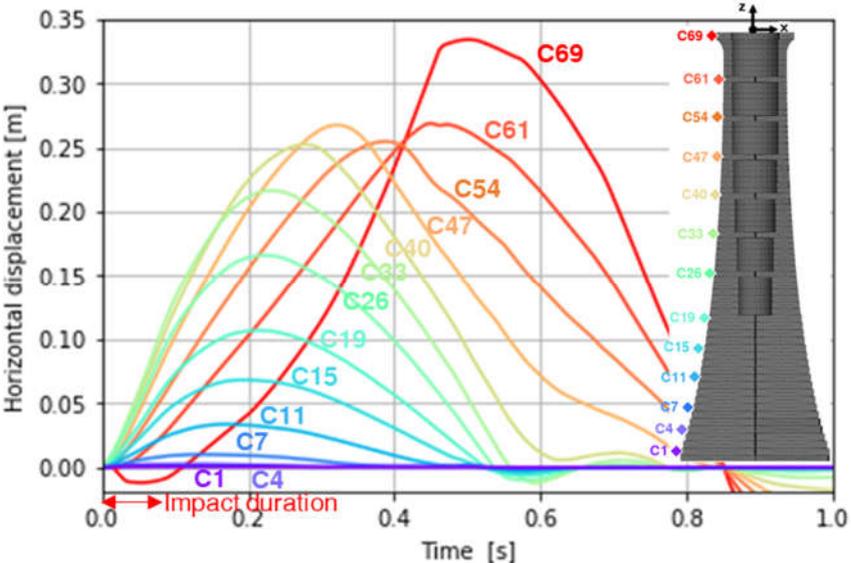


**Figure 10. Maximum horizontal and vertical displacement from DEM analyses together with limit analysis threshold values; range of normalised impact forces 0.01 – 0.50**

In order to explain the change from linear to parabolic trend for the maximum displacements, it is worth comparing the structural response for a strong impact, i.e. the 50 years return period wave, with the response for a weak generic wave of normalised peak force equal to 0.01. The structural response in terms of vertical and horizontal displacements of the DEM model for the estimated impact of the 50 years wave is shown in Figure 11 and Figure 12 respectively. This is a particularly intense impact with normalised force equal to 4.94. The results yield intense rocking and opening of the horizontal joints. The response time-histories reveal a time lag of the peak displacements between the upper and lower part of the lighthouse. The vertical displacements reveal that the joint opening is initiated between the control points C7 and C19 (Figure 11). The maximum uplift is found on the highest control point, i.e. C69, and is recorded around 0.28 s after a first peak found on the lower courses. Similarly, the maximum horizontal response is equal to 0.14 m and is found at C69 with a time lag of 0.2 s compared to a peak recorded on the lower courses (Figure 12).

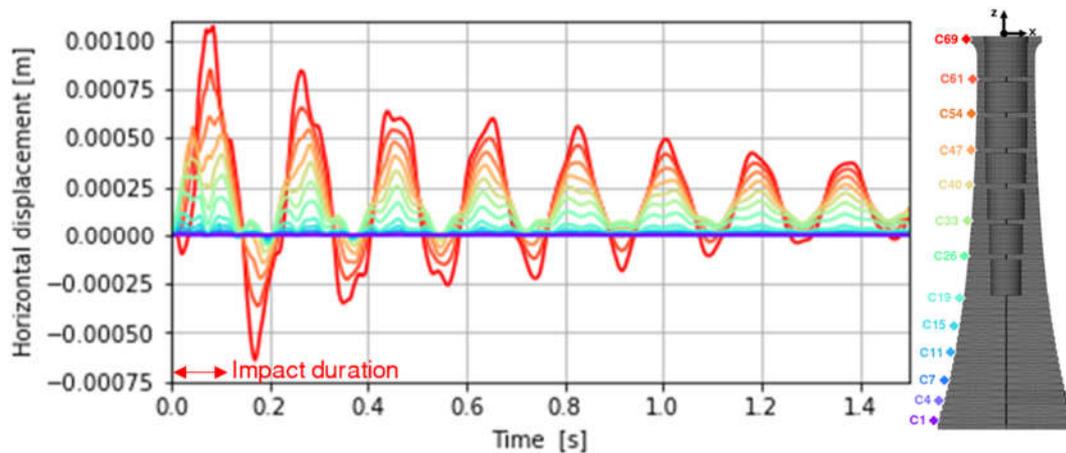


**Figure 11. Structural response of the DEM model for the wave of 50 years return period recorded at the control points: vertical displacement**



**Figure 12. Structural response of the DEM model for the wave of 50 years return period recorded at the control points: horizontal displacement**

The structural response of the weak impact is presented in Figure 13. This graph shows the horizontal displacement of the control points for a duration of 1.5 s, which includes the impact time (initial 0.075 s) and a damped post-impact free-vibration. It is worth noticing the intense phase difference of the upper versus the lower control points for the beginning of the motion. The higher frequencies (dominating in the lower courses) though are gradually damped out and all areas of the lighthouse pass to an in-phase vibration. Compared to the intense rocking caused by the 50 years wave, this behaviour resembles the dynamic oscillation of a quasi-linear elastic structure. Therefore, this qualitative comparison suggests that the structural response of the lighthouse passes from quasi-linear elastic oscillation to highly nonlinear rocking for increasing wave impact intensity.



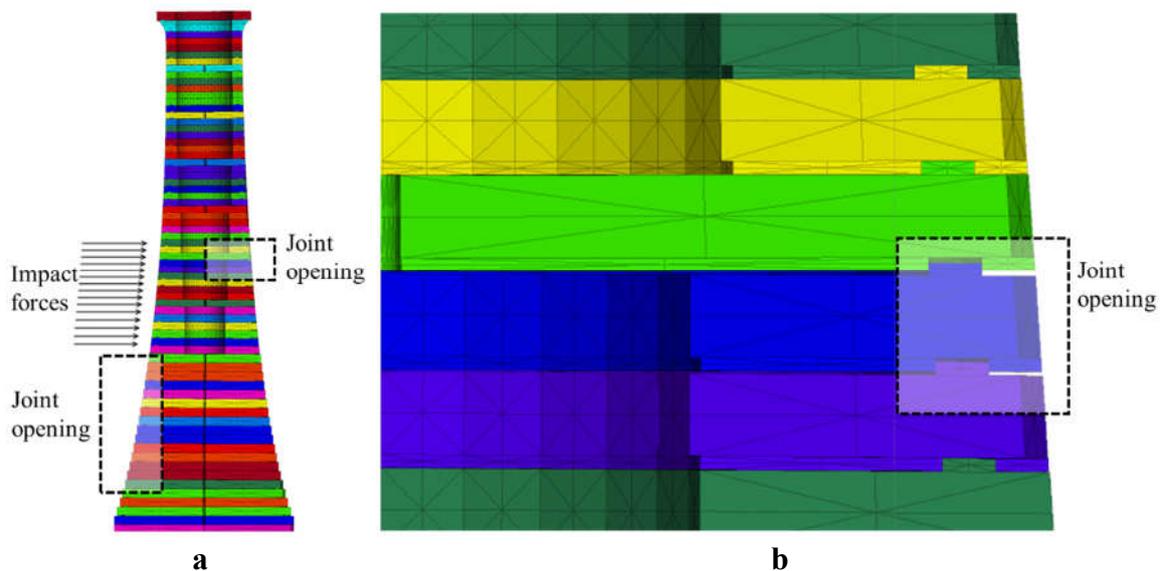
**Figure 13. Horizontal displacement at control points for normalised wave impact force equal to 0.01**

### Structural assessment for the 50 years wave impact

According to the DEM results for the 50 years return period wave impact, the maximum horizontal displacement calculated on the top of the masonry body (control point C69) is equal to 0.33 m. For the same point, the maximum uplift is equal to 0.14 m, which is around twice the height of the vertical key. However, due to the elasticity of the structure, accounted by the stiffness of the joints, this uplift is not concentrated at a single joint, which would be catastrophic, but is a sum of multiple joint separations. The opening of joints is concentrated on two areas of the lighthouse (Figure 14a). The first involves the 4<sup>th</sup> till the 19<sup>th</sup> course, below the impact, where the joints open on the side of the impact. The second area is more localised and consists of the 31<sup>st</sup> till the 33<sup>rd</sup> course, just above the resultant of the impact forces, which open on the opposite side of the impact (Figure 14b). On the impact side, the maximum vertical separation between successive courses of stones is equal to 0.015 m and is recorded between the 9<sup>th</sup> and 10<sup>th</sup> course. This uplift is smaller than the height of the vertical keys, hence sliding is unlikely to happen in this area. On the opposite side, the maximum vertical separation between successive courses of stones is equal to 0.032 m and is recorded between the 32<sup>nd</sup> and 33<sup>rd</sup> course. Although sliding forces are not applied to this area, this uplift corresponds almost half of the key height. This suggests that the structural response is particularly intense.

According to limit analysis for the 360° section, joint opening on the impact side takes place at the joints below the impact forces. Once the overturning mechanism has been activated, accelerations due to inertial forces are applied to the rotating body, i.e. the courses above the open joints, which can cause new joint openings and overturning on the opposite direction. The same

Python code that was created for the limit analysis was enhanced in order to predict joint opening on both sides of the structure. The code predicts opening between the 13<sup>th</sup> and 20<sup>th</sup> course on the impact side and the 25<sup>th</sup> and 26<sup>th</sup> course on the opposite side. Although these openings are at the same regions as the ones predicted by the DEM analysis, the two methods do not perfectly align. This is expectable due to parameters such as damping, friction at the keys, and mainly the elasticity that are taken into account only in the DEM model. The elasticity of the structure allows limited deformation and therefore redistribution of the internal forces that influence the structural response by slightly shifting or expanding the joint opening areas.



**Figure 14. Joint opening at 0.075 s for the 50 years wave impact: (a) overall aspect; (b) detailed view between the 31<sup>st</sup> and 33<sup>rd</sup> joint**

## Conclusions

The coupling of limit analysis with DEM time-history analyses for wave impacts offers valuable knowledge about the structural behaviour of the Wolf Rock lighthouse. The main findings of this paper are summarised below:

- The iterative code developed for the limit analysis, offers important understanding about the structural behaviour of the lighthouse. The limit analysis results show that the combination of bigger diameter and greater weight near the bottom makes the lighthouse able to resist significantly bigger forces if the impact area is near the bottom. Moreover, the huge importance of the vertical keying to the stability of the lighthouse is revealed. Although an overturning mechanism can be reversible, meaning that exceeding of the threshold does not necessarily mean damage, the sliding mechanism is not reversible.
- The DEM analysis produces very satisfactory results and is able to reveal the crucial areas of the structure where opening of horizontal joints takes place. The same qualitative structural behaviour is shown with the iterative code for the limit analysis. Though in similar areas, the joint opening is not identical for the two methods. This is mainly because of the elasticity of the DEM model that changes the distribution of the internal forces.
- The structural response of the lighthouse resembles a quasi-linear elastic vibration for small impacts and becomes highly nonlinear with intense rocking and joint separation for stronger impacts. Moreover, the parametric analysis shows that the amplitude of the

maximum vertical and horizontal displacements follows a linear trend with for small impacts and gradually becomes parabolic for stronger impacts. This transition is not strictly correlated with any of the limit analysis thresholds. For the most conservative limit analysis assumption, i.e. overturning section of 60°, the trend is still linear and becomes parabolic before the exceedance of for the least conservative threshold, i.e. 360° section.

- The 50 years wave impact that was calculated based on the climatic wave conditions for Wolf Rock which has a steep bottom topography, causes intense rocking and opening of the horizontal joints. This wave causes joint opening both on the side of the impact, below the impact area, and on the opposite side, above the impact area. The maximum separation between successive joints is 0.032 m which is roughly half the height of the vertical keys. A maximum horizontal displacement equal to 0.33 m and uplift equal to 0.14 m are found for the control point on the top of the masonry body. Both the limit analysis and the DEM analysis confirm that this is a particularly intense wave. Wolf Rock lighthouse would not be able to survive such an impact without the presence of vertical keying.

## Acknowledgments

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