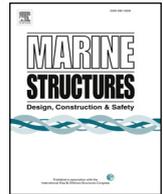




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## Marine Structures

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## Experimental modal analysis of British rock lighthouses

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## ARTICLE INFO

## Keywords:

Lighthouse condition assessment system  
Identification  
Forced vibration  
Operational modal analysis

## ABSTRACT

Iconic lighthouses constructed on offshore reefs around the British Isles in the 19th century continue to play a crucial role in safe navigation, but the longevity of these historical structures is threatened by extreme weather. A program of experimental dynamic investigations has been carried out to support characterisation of extreme impulsive breaking wave loads on these structures, using monitored response data. This paper describes the procedures and outcomes of this program, which included modal tests of a collection of six of these lighthouses between June 2016 and October 2017.

Five of the six lighthouses tested (Les Hanois, Wolf Rock, Longships, Bishop Rock and Eddystone) feature a 20th century metal helideck atop a 19th century masonry tower, with a Scottish lighthouse (Dubh Artach) being the exception that provides baseline behaviour of a relatively simple tower. All the masonry towers are imperfectly axisymmetric to some degree and all present logistical challenges for experimental work as they can only be accessed by helicopter flights subject to severe weather and time constraints. Against such challenges it was possible to identify key modal parameters, and to highlight some interesting effects due to symmetry and helideck retrofit.

Notable findings were that most important modes have frequencies ranging between 4 Hz and 7 Hz and modal masses as low as ~200 t. The rarely investigated effect of imperfect axisymmetry on forced vibration testing is studied, along with the introduction of additional modes due to retrofitted helideck. The implications of these effects on experimental modal analysis from forced vibration test data is illustrated.

Finally, accelerations recorded on Wolf Rock Lighthouse during the 2017–2018 winter storm season show the modal test data can be used to infer breaking wave modal impulses up to 8 kNs.

## 1. Introduction

Victorian era rock lighthouses remain a vital aid to maritime navigation, yet the severe environmental loads they endure are not understood. Funded by the UK Engineering and Physical Sciences Research Council (EPSRC), Project STORMLAMP was initiated to develop a combination of physical and numerical simulation tools for both loading and structures needing to be validated by direct measurements of full-scale performance.

The direct measurements are described in this paper and feature lighthouses located on offshore rock outcrops or reefs which

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<https://doi.org/10.1016/j.marstruc.2018.07.001>

Received 15 March 2018; Received in revised form 18 June 2018; Accepted 12 July 2018

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represent hazards to marine navigation in the English Channel and Western Approaches. The numerous shipwrecks in this area and around the British Isles led to the incorporation of Trinity House in 1514 by royal charter from Henry VIII. Although Trinity House built its first lighthouse in Lowestoft in 1609, the first *rock* lighthouse was constructed on Eddystone Rocks south of Plymouth in 1698 and survived until the Great Storm of 1703 when it was destroyed along with its designer. The present Eddystone Lighthouse is in fact the fourth structure to be built on the site, the third having been rebuilt on Plymouth Hoe.

Likewise, the present structures at Wolf Rock, Longships and Bishop Rock (off the south west of tip of Cornwall) are successors learning from failures of previous designs, although the structures at Les Hanois (Guernsey) and Dubh Artach (Scotland) are the original structures, all of them constructed in the 19th century. With design evolution these British structures have, with the exception of Bishop Rock that was reinforced between 1882 and 1887, functioned with only minor structural repairs to their super-structures to the present day. Condition assessment has been mainly limited to visual checks on the masonry structure, with more detailed assessment of foundation, helideck and interior.

Information on condition assessments of lighthouses has been obtained with assistance from relevant lighthouse authorities. In the UK these are Trinity House, the Northern Lighthouse Board and Irish Lights, all operating under the umbrella of the General Lighthouse Authorities (GLAs). Blakeley and Warke [1] produced a Task Report for the UK GLAs on building condition monitoring, focusing on issues of humidity and condensation, and their effects on stonework. Their report built upon earlier work undertaken prior to solar photovoltaic installations [2] when the structures would have experienced different internal conditioning regimes. Warke et al. [3] present results of an associated monitoring study on the interior stonework of two Irish Lights towers, describing the different temporal and spatial characteristics of the deterioration. In 2009/2010 Trinity House commissioned full structural surveys of the rock lighthouses described in this paper, which comprised inspections of the interior and exterior including the helidecks and landing areas, generally noting that the structures were in reasonable condition. Raby et al. [4] describe a subsequent pilot project on the Eddystone Lighthouse that developed a monitoring system to relate the wave loading to the structural response of the tower. The structural response information, useful for assessing the tower's condition, led to the present STORMLAMP project to investigate a number of UK GLA rock lighthouses: Les Hanois, Longships, Wolf Rock, Bishop Rock and Eddystone which are managed by Trinity House, Fastnet which is managed by Irish Lights and Dubh Artach which is managed by Northern Lighthouse Board. Condition assessments through dynamic testing of all but Fastnet Lighthouse between June 2016 and October 2017 are described in this paper. Work on Fastnet is reported as part of a different study including detailed numerical and physical modelling [5].

This paper describes an experimental campaign to assess the condition of the lighthouses and provide information to support estimation of the severe loads they experience due to breaking and broken wave impacts. Structural characteristics have been identified as modal parameters via full-scale dynamic testing and linked modal analysis. The parameters, which are the modal frequencies, masses and shapes can support estimation of wave loads from measurements of their structural response through long-term monitoring.

The paper begins with structural descriptions of the lighthouses tested, with a focus on Wolf Rock, for which the most useful historic account is available and which (based on the modal test data) was chosen for long-term monitoring. Next, a relatively detailed description of the first vibration test, of Les Hanois Lighthouse, is presented. Modal test results from the set of lighthouses are summarised, pointing to effects on dynamics of imperfect symmetry and the retrofitted helidecks. These effects are then investigated using simple mathematical simulations related to the experimental data. Finally, a snapshot of monitoring data from Wolf Rock during Winter 2017/2018 is presented and interpreted, based on the modal test data, providing initial direct estimates of impulsive wave loads.

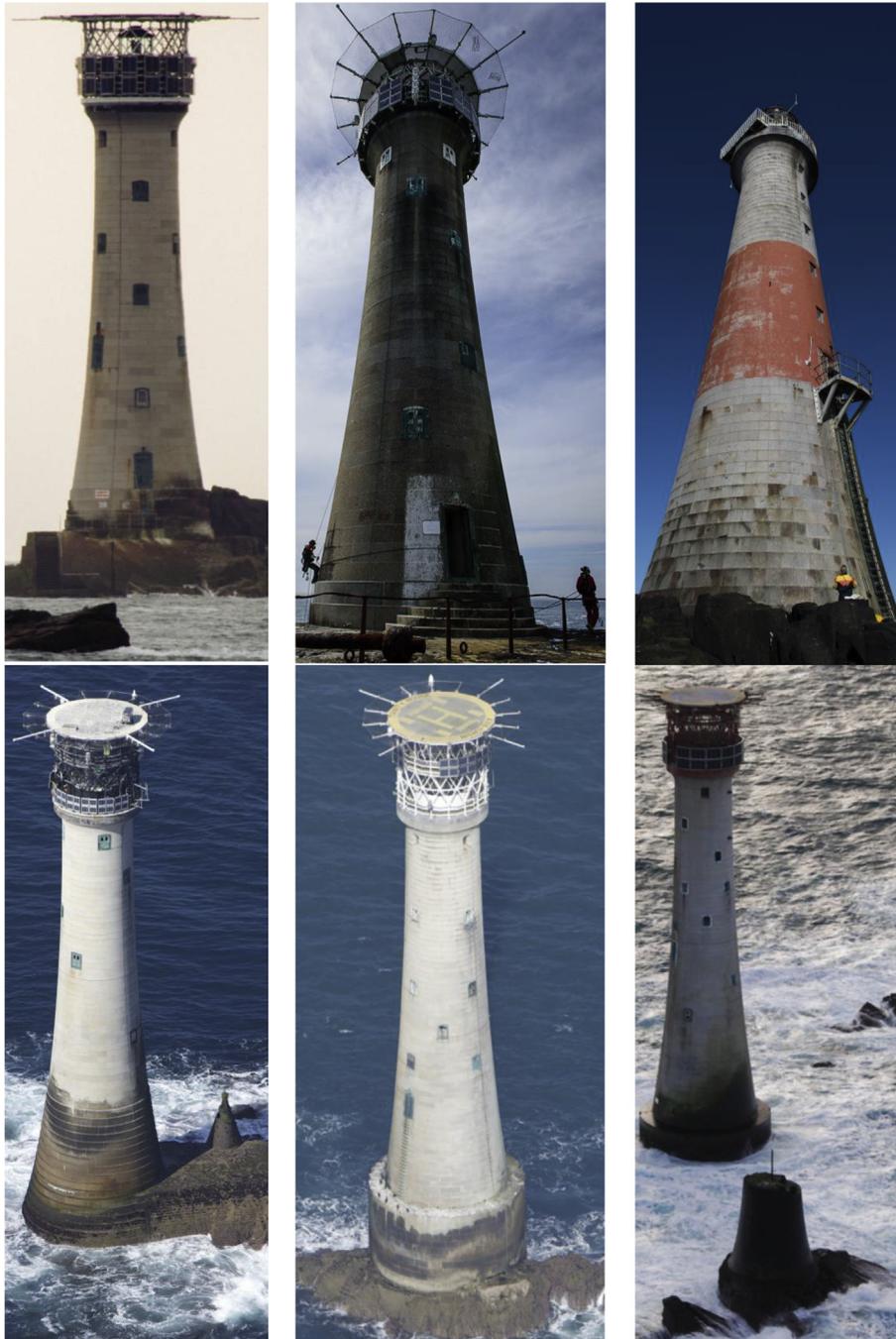
## 2. Description and structural details of the lighthouses

Details on location, construction and testing of the six lighthouses are given in Table 1. Five helideck-equipped lighthouses are studied along with one example with no helideck, Fig. 1.

There are various sources of information available on the construction of these lighthouses including numerous internet sites, drawings and other materials archived by the UK GLAs, and popular books e.g. Ref. [6]. Primary sources are relatively rare and include papers presented at the Institution of Civil Engineers [7–9] describing construction of Eddystone, Bishop Rock and Wolf Rock

**Table 1**  
Lighthouse and modal test details.

Lighthouse	Location	Built	Designer	Tower height	Test date
Les Hanois	Guernsey west coast, 49°26'06.2"N 2°42'08.4"W	1860–1862	James Douglass	36 m	2 Jun. 2016
Wolf Rock	15 km west of Land's End, 49°56.72'N 05°48.50'W	1861–1869	James Walker	41 m	18 Jul. 2016
Longships	2 km west of Land's End, 50°4'00.69"N 5°44'48.39"W	1869–1875	James Douglass	35 m	23 Aug. 2016
Bishop Rock	South west of Scilly Isles, 49°52'22.52"N 06°26'44.49"W	1851–1858; 1881–1887	James Walker	49 m	26 Sep. 2016
Dubh Artach (no helideck)	West of Oban, 56°07'56.76"N 06°38'4.74"W	1868–1872	Thomas & David Stevenson	33 m	8–9 May 2017
Eddystone	21 km south west of Plymouth, 50°10'48"N 4°15'54"W	1879–1882	James Douglass	49 m	10–11 Oct. 2017



**Fig. 1.** Rock lighthouses around the British Isles. Upper row from left: Les Hanois, Longships, Dubh Artach (author photos). Lower row from left: Wolf Rock, Bishop Rock and Eddystone (Lower row photos courtesy of Trinity House).

lighthouses whose elevations are shown in Fig. 2 along with details of sample courses (levels) of granite blocks. Note that the quality of the drawings is imperfect due to their age, indicated by Douglass' signature on the Eddystone course detail.

These structures share common major characteristics of having been constructed from pre-shaped interlocking granite blocks and all but Dubh Artach have been retrofitted with a helideck, fixed to the top of the masonry tower and straddling the lantern. While Dubh Artach features a monotonic taper for most of its height, the other five lighthouses take the form of a concave elliptic frustum first introduced by John Smeaton in the third Eddystone Lighthouse.

The technique of lighthouse construction is described for Wolf Rock based on the detailed description by James Douglass in Ref. [9], with similar techniques applied to the other structures. All dimensions originally quoted in imperial units (usually as integer

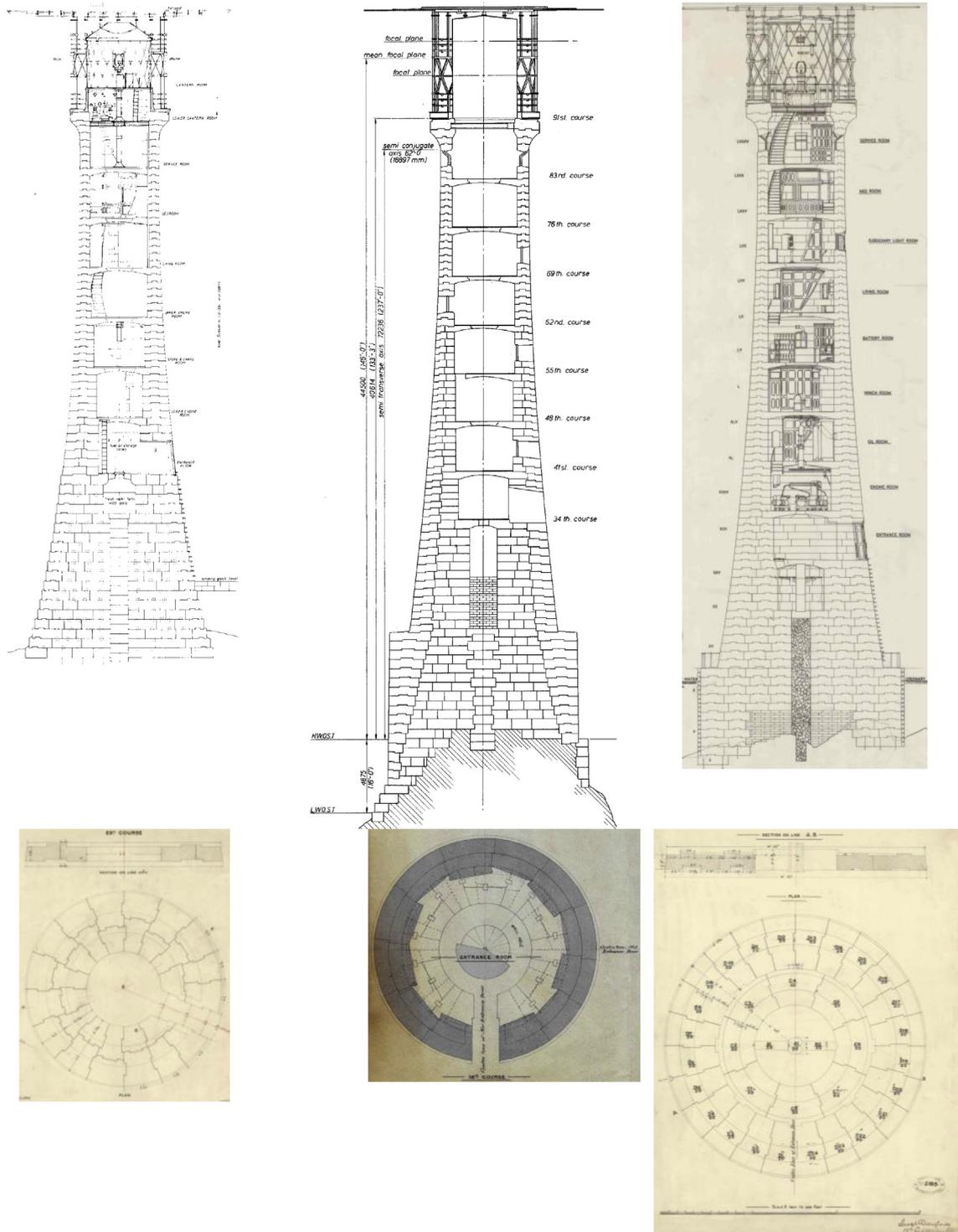


Fig. 2. Wolf Rock, Bishop Rock and Eddystone lighthouse elevations (above) and details of sample courses (below). Wolf Rock 23rd course and Bishop Rock 23rd courses are at entrance level, Eddystone 20th course is at sump tank level. Common scale applies among elevations, separate common scale among courses. All drawings courtesy of Trinity House.

values) have been converted to SI and rounded.

Wolf Rock lies 15 km from the far south west tip of the English mainland, with 36.6 m deep water close to the reef on all sides but the south east. A beacon was first erected on Wolf Rock in 1795 but was soon destroyed by storms. A stone structure was considered

in 1823 but the cost (£150,000 at the time, more than double the final cost of Eddystone) was considered prohibitive. Instead, a new beacon was erected, at a cost of £11,300 over a period of five years during which time only 302 h of work was possible on the rocks. The hollow conical steel support (Fig. 1) still survives but the various masts were lost to the waves, and construction of the present granite tower began in 1862 after a brief survey by Douglass in 1861. Time available on the rock for construction work was highly dependent on weather, so that 83 h were possible in 1862 and 206.5 h in 1863, enough for blasting and cutting of the shallow foundation pit and for the landing stage to be 'half completed'. By the end of 1864 the landing platform (comprising  $0.61 \text{ m} \times 0.3 \text{ m} \times 0.15 \text{ m}$  thick keyed granite ashlar blocks) was complete and the first masonry course of the tower was in place.

The tower was constructed as 70 courses of granite blocks, with masses typically in the range 2–4 t, quarried and pre-shaped before being shipped to site. The 1st course is only two blocks and the 2nd course is a complete circular disk of blocks each secured to the rock below by 50 mm diameter Muntz metal bolts sunk 0.3 m into the rock and fox-wedged at each end. Muntz Metal is a brass alloy of 60% copper, 40% zinc and a trace of iron, and fox-wedging involves driving a shaft through a block and onto a wedge set on the end face of a recess that broadens with depth, so that the shaft is split at the end and mechanically locks the block in place. Face stones in the external ring of blocks are dovetailed vertically and horizontally to fit vertical and circumferential neighbours, and in addition each stone in the 3rd to 20th courses is secured to the course below by two 50 mm galvanised puddled steel bolts sunk 0.23 m into the lower course. Dovetailing of the face stones is achieved by a 76 mm raised dovetail band on the upper surface and end, with corresponding dovetailed recess cut in the adjoining stones, with dimensions just allowing the raised bands to enter the recesses freely when setting. To anchor the blocks and prevent water ingress Medina Roman cement was used between the blocks up to the level of high water spring tides, and Portland cement was used above high water. Experiments on a pair of blocks joined this way using Portland cement showed the result to have strength equal to solid granite.

1st to 20th courses are 0.61 m thick, with thickness changing at each level to a minimum 0.46 m for courses 49–67. Outside diameter (OD) reduces from a maximum 12.23 m at the complete 2nd course to a minimum 5.18 m 'at the springing of the curve of the cavetto under the lantern gallery' in course 68. Internal diameter (ID) increases with level; 2.74 m for entrance room, 3.05 m for lower engine room, 3.35 m for 'store and crane', 3.66 m for upper engine room including roof then 3.81 m for living room, bedroom and service room. The lantern gallery course 70 that supports the lantern structure has 3.96 m and 6.4 m OD. The total volume of granite is  $1260 \text{ m}^3$  having mass 3350 t, and the total cost of the lighthouse was £62,726. The mass of the original lantern machinery and enclosure (glazing and roof) is not known, neither is that of the present system.

Wolf Rock was, in 1973, the first UK lighthouse to be equipped with a helideck, which comprises a steel frame of square and rectangular hollow sections. The 6.35 m tall frame comprises a set of 2.13 m vertical gallery posts, a ring of 1.52 m tall cruciform braces and upper posts supporting a grillage of horizontal elements. This grillage holds aluminium infill panels that distribute downward live loads (helicopter, freight etc.) to the frame, although vertical forces generated by wave impacts can be strong enough to break shear pins holding the panels in place and launch them into the sea.

Welded assemblies of the frame (e.g. cruciform units) were bolted together on site. They are structurally separate from the lantern enclosure but attached to the gallery at the top of the tower via a reinforced concrete circumferential slab up to 0.2 m thick with ID 4.57 m, OD 6.55 m. The total mass of the helipad landing area (grillage, rings and outriggers plus infill panels) is estimated at 5.6 t, while the mass of the cylindrical vertical frame (vertical posts, horizontal rails, bracing cruciform) is estimated at 5.86 t. Extra (unknown) mass is due to solar panels at gallery level, an intermediate level walkway, ladders and decades of accumulated guano.

Few structural details are available for **Les Hanois** other than the cost (a relatively cheap £25,296) and total volume of granite (including landing stages) at  $695 \text{ m}^3$  [7]. In terms of fixity, as well as employing vertical dovetailing (for the first time) the lighthouse is built more or less symmetrically around a rock outcrop, and Trinity House drawings indicate bolts with unknown anchoring sunk into the rock.

**Longships** is a similar height to Les Hanois but the tower is built around a rock outcrop on the southeast side of the reef. Partial courses are mechanically locked to directly interfacing rock and to each other via a combination of bolts and dovetailing, starting with one to three blocks on courses 1–4, then building almost monotonically from approximately 5/8 of full circumference at 5th course to approximately 5/16 full circumference at the 10th course, with the 11th course covering the entire area and capping the central water tank. Original drawings provided by Trinity House show anchoring of courses interfacing with rock via bolts and fox-wedging, with a combination of dovetailing and bolting between courses up to the 11th. Like Les Hanois, Longships has seven rooms below the lantern gallery, and the helideck vertical framing system is the shortest among the five helideck-equipped lighthouses. The lighthouse cost £43,869 and the granite volume including landing stage is  $1348 \text{ m}^3$ .

**Dubh Artach** is unusual in both shape and lack of helideck, but was the most expensive (£72,524). The stone tower appears to be perfectly axisymmetric apart from the row of openings (windows and door) along a vertical line and the asymmetric balcony. It has six rooms plus a deep gallery level and sub-level with entrance that is presently inaccessible.

**Bishop Rock** is possibly the most unusual lighthouse, the current structure being the third incarnation. The first structure using cast iron piles was swept away in 1850 and construction of a masonry tower in the style of the Smeaton's Eddystone tower began in 1851 and was completed in 1858. Bishop Rock, whose name possibly derives from its shape resembling a bishop's mitre, is a 47 m long 16 m broad granite outcrop that drops sheer and steep to 37 m deep water. The approximately 30.5 m tall tower was known to vibrate during severe storms and some external granite blocks were found to have been split due to vertical strain in the tower. Even after the tower was 'strengthened from top to bottom by bolting heavy internal ties to the internal surfaces of the walls' there was evidence after a violent storm in 1881 of further damage, with 25 kg granite pieces split from the exterior faces, so Trinity House decided to strengthen the existing structure. To do this, an outer casing of granite blocks was added, up to the level of the original service room (below the lantern). These blocks are dovetailed vertically and circumferentially and also keyed into the blocks of the existing tower. Additional blocks in each course up to the 20th were further anchored by 0.46 m Muntz metal bolts fox-wedged

0.23 m into the course below, as was done for the original tower. Like Longships, the lower courses of the original tower do not form a full circle, resulting in asymmetry, although this has been partially corrected by creation of the 12.5 m diameter cylindrical base that rises to 12.2 above the foundation, forming a landing platform. The few gallery courses above the service room of the original lighthouse were removed and four new rooms added (making a total of eight), and the level of the lantern focal plane was increased from 33.5 m to 44.5 m. Bishop Rock is probably the largest and certainly the most massive of the lighthouses presented here, although the mass of the granite is not given by Douglass. The cost of the ‘improved granite lighthouse’ was £64,889.

The helideck was added in 1976 and is similar to the Wolf Rock structure, although somewhat larger, with total mass estimated as 17.4 t, 8.22 t of that being the horizontal steel frame and infill panels of the landing deck. This excludes non-structural components such as the high-level solar panels.

As mentioned earlier, the present **Eddystone** Lighthouse is the fourth structure built on Eddystone Rocks, an outcrop located in the approaches to Plymouth, a port with (at the time of lighthouse construction) growing commercial and military importance. The third structure (Smeaton's Tower) operated successfully until it had to be replaced because the gneiss rock on which it was founded had been undermined at its base and because waves during extreme storms rose ‘considerably above the summit of the lantern’, eclipsing the light. The foundation for the new lighthouse was planned so as to avoid any unnecessary removal of sound rock and no blasting was allowed, so all rock was removed using ‘drills, jumpers, cleaving gear and picks’. Each foundation course face stone was sunk to a depth of not more than 0.3 m below the surrounding rock and these stones were each secured to rock by fox-wedged Muntz metal bolts. Courses 1–5 adapt to the rock shape but are largely axisymmetric, with complete courses starting from the 6th. As for Bishop Rock a cylindrical base is formed, having 13.4 m OD but only 6.7 m high. The tower contains nine rooms, the top seven having ID 4.3 m and height 3.05 m. Roofs for the lower eight rooms are (as with other lighthouses) granite courses covered with slate tiles, with iron stairs and partitions, and gunmetal windows and shutters. According to Douglass, the masonry tower comprises 1846 m<sup>3</sup> of granite, with mass 4911 t, and the lighthouse cost £59,255, 23.5% under budget. The 5.92 m tall helideck vertical structure has mass 6.34 t attached to a set of steel foundation beams bolted onto the gallery and carries solar panels and 0.39 t intermediate walkway. The landing deck (horizontal grillage) itself comprises 3.6 t of steel work and 1.95 t of aluminium infill panels so total mass for the structure is a minimum 12.28 t.

### 3. Test objectives and planning

The modal testing aimed to provide information on the structural condition of these lighthouses delivered in the form of comprehensive sets of modal properties which include the natural frequencies, damping ratios, mode shapes and modal masses. Modal masses are simultaneously most important for relating wave loading to response and particularly difficult to estimate, which is why values are seldom reported.

Superficially the selected lighthouses might be expected to have modal properties similar to industrial chimneys [10] that are almost axisymmetric. This means that fundamental (first) natural frequencies would appear as a pair of close modes with corresponding modes shapes in orthogonal directions having weakly defined alignment in ‘principal axes’ strongly affected by structural details that disrupt the axisymmetry. For example, in Fig. 1, the various windows and entrances of the lighthouse are external signs of disrupted symmetry, while internally opening in (masonry) floors for the spiral staircases and arrangement of heavy items such as fluids, batteries or generators could have an effect. While Wolf Rock, Dubh Artach and Eddystone lighthouses are more or less axisymmetric in the lowest stone courses, the Wolf Rock landing stage could affect symmetry, and Les Hanois, Longships and Bishop Rock have clear asymmetry by adaptation of the lower granite courses to the reef shapes.

The aim for the modal testing was thus to identify at least the pair of fundamental modes most susceptible to breaking wave loading, plus as many higher modes as possible, along with their mode shapes, damping ratios and modal masses.

The choice of test methodology was between ambient vibration testing requiring only a set of accelerometers and recorder, and forced vibration test which in addition requires a mechanical shaker and power amplifier. The only information to inform the decision was data from Eddystone Lighthouse [4] where the first mode was indicated at 4.4 Hz. Moreover archive structural drawings providing information about the mass distribution suggested that in ideal conditions of calm weather experimental modal analysis using an electro-dynamic shaker with force output around 150 N would be feasible. Hence the modal test procedures were designed taking a gamble that forced vibration testing would work against an unknown background of ambient vibration induced by onboard machinery, wind and wave loads.

Although landing stages for boat access exist at most of the lighthouses, the only access permitted for the measurements was by helicopter, and the modal tests sometimes required two helicopter return trips from the nearest heliport to ship out and two to ship back. There were limits to weight of equipment (and personnel) to be shipped, including provision for planned or unplanned (due to ‘weather’) overnight stay. With the shaker and amplifier taking the bulk of the weight allowance the weight of cabling and sensors was minimised and (typically) 12 accelerometers were used, arranged in multiple setups to cover all available levels of the lighthouse from entrance to helideck in orthogonal horizontal directions.

The detailed methodology evolved at first quickly to obtain best outcomes with given constraints (mainly lack of time) then stabilised. The last test (of Eddystone) probably produced the best quality results partly due to lack of extreme time pressure. The first test, of Les Hanois, is described in most detail as it largely set the pattern for future tests and revealed the main challenges to experimental modal analysis of these structures.

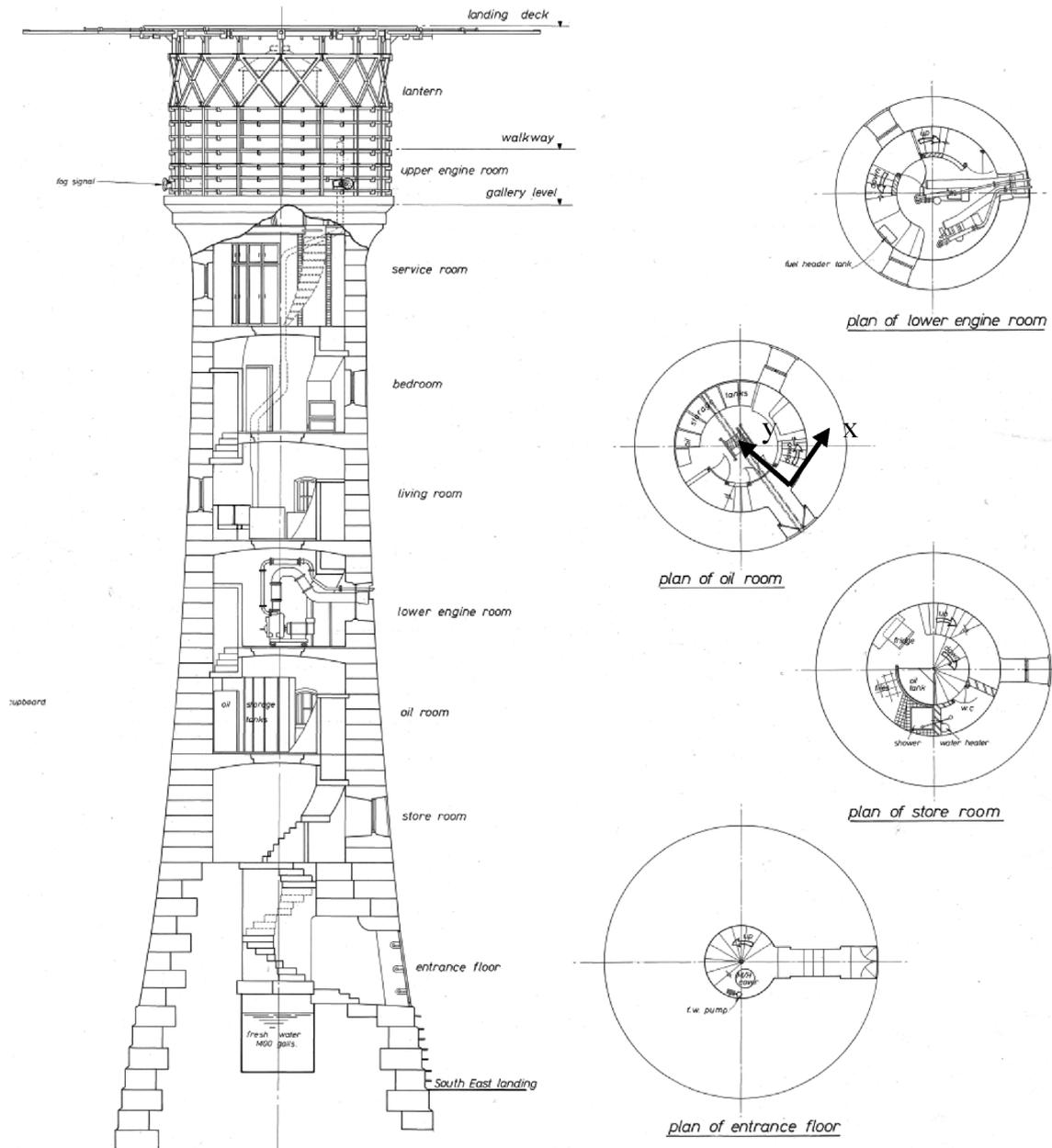


Fig. 3. Les Hanois Lighthouse arrangement. Level numbers L1-L11 are indicated to the left. Arrows on plan of oil room (L3) indicate plan location and (x,y) alignment of accelerometers at all levels.

#### 4. Les Hanois lighthouse modal test

Taking the first opportunity in the schedule of Trinity House maintenance visits, Les Hanois was tested first. There was no chance for reconnaissance, so it represented a very steep learning curve and the process is described in some detail as an exemplar.

The lighthouse was accessed by a three-person test crew in a single airlift by a short flight from Guernsey Airport by Eurocopter EC135. Planned departure was 09:00 1st June 2016 to provide a full day of testing on site, but due to limited visibility the departure was delayed, with arrival on station at 13:10 on 2nd June. As well as weather restrictions, access to lighthouses is limited by available overnight accommodation in case poor weather prevents return flights by visual flight rules. A Trinity House maintenance crew was on Les Hanois using all available accommodation, requiring the test crew to return the same day at 18:00. Allowing for equipment transfer between helideck and lighthouse interior, unpacking, setting up and repacking meant that setting up and testing was limited to a window of approximately 3 h. Even without equipment glitches, the lack of reconnaissance and the very tight time schedule limited modal test activity to only absolutely essential activities.

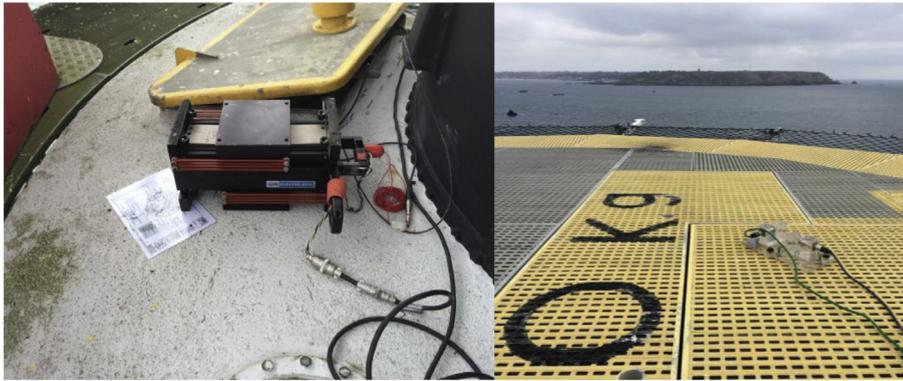


Fig. 4. Les Hanois modal survey. Shaker at lantern walkway, L9 (left) and accelerometers on helideck, L10 (right).

The ten levels for measurement numbered L1 through L10 are shown in Fig. 3 and include the steel helideck (L10) and 'lantern' (L9) which is the steel structure sitting on the gallery atop the masonry tower. The shaker and pair of orthogonal reference accelerometers were set on the steel external walkway at L9 that also provided access to the lighthouse from the helideck, where a second pair of accelerometers was located, as shown in Fig. 4.

Descending in to the masonry tower comes first the service room (formerly upper engine room, L8) with floor set approximately 0.5 m below the top of the gallery. This level was used for setting up data acquisition equipment. Then descending the tower (by spiral staircase on the internal granite wall) come the service room (L7), bedroom (L6), living room (L5), lower engine room (diesel generator, L4), oil room (L3), store room (with shower and toilet, L2) and entrance (L1) for emergency access to the reef.

For the first of two arrangements of accelerometers (setup 1) eight accelerometers were arranged as orthogonal pairs at L1, L2, L5 and L6, a total of 12 sensors including those at L9 and L10. For all but L10 the Honeywell QA-750 quartz-flex accelerometers were arranged at the inner wall of the masonry tower at the same compass bearing with respect to the lighthouse vertical axis, with the y axis always pointing to the central (vertical) axis of the tower. This method of positioning and alignment was used on all levels of all the lighthouses tested and made use of recurring reference features such as corners of windows and a T-shaped wooden jig. Where inner wall radii are the same (e.g. for L2-L7 at Les Hanois) accelerometer locations lie on a vertical line. For the helideck the accelerometers were arranged at the safest and most convenient location, while having the required compass bearings.

For Les Hanois x was positive in north east direction and y was positive in north west direction. These directions are indicated, for L3, in Fig. 3.

For the second arrangement (setup 2), accelerometers at L1 and L2 were moved to L3 and L4, and those at L5 and L6 to L7 and L8. The aim of using two setups with orthogonal accelerometers was that the two reference pairs at L9 and L10 would enable assembly of mode shape pieces [11] in the modal analysis process.

For each setup, the plan was for a minimum of three measurements:

- Ambient response with no shaker operation
- Forced vibration with shaker aligned in the x direction
- Forced vibration with shaker aligned in the y direction.

Accelerometers were connected using an optimised procedure using short color-coded single channel cables and long four-channel cable reels, all having robust connectors. Even so, positioning the accelerometers, laying and tidying the signal cables and checking connections took about 1 h. Relocating accelerometers between levels and re-aligning them between setup 1 and setup 2 also took a considerable amount of the short time available on the lighthouse.

Data acquisition used a Data Physics spectrum analyser acquiring at 204.8 Hz. Measurements are summarised in Table 2 that excludes test runs for system checks and accounts for 70.5 min of measurements.

**Table 2**  
Les Hanois Lighthouse measurement sequence.

Run	Setup	levels (Fig. 3)	Shaker direction	Excitation	Duration/s	comment
5	1	L1 L2 L5 L6 L9 L10	–	ambient	940	high frequency noise
6	1	L1 L2 L5 L6 L9 L10	y	swept sine 3–20 Hz	690	good data
7	1	L1 L2 L5 L6 L9 L10	x	swept sine 3–20 Hz	640	good data
13	2	L3 L4 L7 L8 L9 L10	–	ambient	340	much high frequency noise
14	2	L3 L4 L7 L8 L9 L10	x	swept sine 3–20 Hz	720	good data
17	2	L3 L4 L7 L8 L9 L10	y	swept sine 3–20 Hz	600	good data
19	2	L9 L10	y	swept sine 5–6 Hz	300	good data

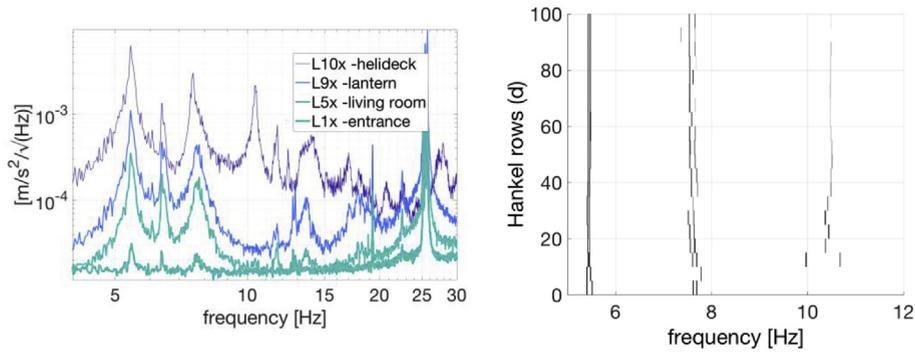


Fig. 5. (Left) Les Hanois ambient response auto-spectra, with legend indicating sensor location and orientation, and (right) OMA output.

#### 4.1. Ambient response and operational modal analysis (OMA)

The first measurement, run 5, with ambient excitation was used to check the frequency range for the shaker excitation. Signals were acquired in units of gravity (g) and converted to engineering units where appropriate. Winds on the day for St Peter Port (closest weather station with available data) were 13–19 km/h.

The power spectral density of run 5x direction signals (Fig. 5) shows a strong signal around 25 Hz that is clearly related to the mains power supply, as there are also strong lines at 50 Hz and 75 Hz and weaker lines at 12.5 Hz. All lighthouse electrical equipment is powered from a substantial battery array charged by photovoltaic cells, but when the lighthouse is periodically manned (by a maintenance crew) a generator is used to support the extra load and the result is a combination of mechanical and electrical noise in the response data. The generator has nominal speed 1500 revolutions per minute which can vary (audibly) by up to 10% according to load and this results in multiple sharp spectral lines around 25 Hz. Also, strength of this signal varies strongly between lighthouse levels suggesting a non-resonant structural response to harmonic mechanical excitation by the generator.

Fig. 5 shows four apparent modes in the 5–11 Hz range, with the highest of these appearing only in the helideck. For the modes below 8 Hz the helideck exhibits response an order of magnitude larger than the masonry (granite) tower.

Based on experience with a number of cable-supported bridges [12] and high-rise buildings [13] where artificial excitation is not feasible, operational modal analysis (OMA) was expected to be a viable method for system identification of the lighthouse modal parameters. OMA is a class of system identification methodologies that can be used when artificial excitation is not available but response data are, so OMA is also known as ‘output only’ modal analysis.

A wide range of techniques are available for OMA [14], the more popular of which are currently ‘enhanced frequency domain decomposition’ (EFDD) [15] in various flavours and ‘stochastic subspace identification’ (SSI) [16]. EFDD works on cross-power spectra matrices while SSI works directly on the original time series. Another method still widely used is the ‘natural excitation technique/eigensystem realisation algorithm’ (NExT/ERA) [17] which uses either cross-power matrices or original time series in the NExT step as input to the time domain ERA identification process which is similar in concept to SSI. For ERA, the initial step is to create a ‘Hankel matrix’ whose elements are time instances of the cross-covariance matrices representing impulse response functions of the system. The identification results vary according to the number (e.g. rows) of these sub-matrices, rather like selecting ranges of an exponentially decaying time-domain sinusoidal response to an initial condition. Fig. 5 shows this effect, clearly demonstrating a pair of very close modes below 6 Hz and a less distinct pair below 8 Hz.

Ideally OMA would identify a set of vibration modes with mode shapes defined at all levels and with both x and y components. By using two setups sharing four common degrees of freedom (DOFs), specifically L9 and L10 in both x and y directions, it should in principle be possible to identify an unambiguous set of modes and assemble mode shapes by combining data from the two setups. EFDD and NExT do this by normalising cross-spectra to reference DOFs and carrying out identification on a combined dataset. As SSI operates in the time domain, separate identification may be required for each setup with merging via text files.

OMA in various forms and variants and with different settings was tried, with variable results. The most convincing set of modes, obtained by NExT/ERA combining both setups is shown in Fig. 6 (left). For the figure, the modes are normalised to unity maximum then projected onto their best fit vertical plane which is rotated to an elevation view. The pattern of modal ordinates of different sign between the top of the masonry structure and the helideck for certain modes recurs among all helideck equipped lighthouse and is studied in detail in a later section. The plot does not reveal that the orientation (compass bearing) in the identified modal ordinates sometimes varies significantly between levels. Within the masonry tower the modal ordinate is the largest at the gallery where sensors for permanent monitoring system would be located, hence NExT/ERA was repeated on x and y signals for this location only, using run 5 data. The result is shown in Fig. 6 (right) as a plan view for the first four modes, which appear to align almost as orthogonal pairs. The modal parameter estimates compared differ from those in the left view due to the different mix of data used.

OMA using these traditional techniques proved to be problematic for all six lighthouses due to lack of repeatability, missed modes and unfeasible mode shapes. As it also cannot identify modal mass, most of the effort in the subsequent tests focused on obtaining good quality data from shaker testing for experimental modal analysis, and only these results are reported.

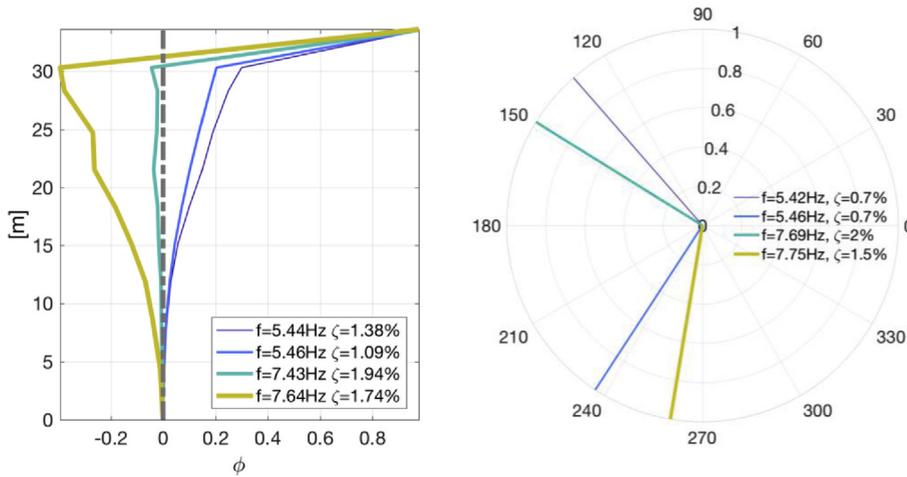


Fig. 6. NExT/ERA on Les Hanois data for both setups projected to a common elevation view (left) and in plan (right) for single setup using only L9 data.

4.2. Forced vibration testing

The difficulty in identifying modal properties purely from ambient data validated the decision to use force vibration testing for all the structures, and the experience influenced test procedure in subsequent lighthouse deployments.

Forced vibration testing is usually applied to floors and footbridges [18,19] using multiple large shakers e.g. the 450 N capacity APS 400, however at 92 kg the APS 400 is not safe to transport to, and manhandle around, a lighthouse. The more manageable 52 kg APS 113 was thus used for all measurements. This comes in low force (LF: 133 N) and high force (HF: 186 N) variants and the latter was used at Les Hanois. A swept sine (chirp) excitation was used to maximise signal to noise ratio (SNR) with sweep up from 3 Hz to 20 Hz in  $T = 40$  s and repeating several times to improve SNR via averaging. The range 3–20 Hz was chosen based on the ambient response spectra (Fig. 5) to cover apparent vibration modes, and proved to be a successful approach for Les Hanois. The 3–20 Hz sweep was repeated with the shaker arranged alternately in the nominal x and y directions for each setup, so as to identify modal properties including modal mass.

The frequency response function (FRF) of Fig. 7 calculated from force and response data using the H1 estimator [20] clearly shows two strong structural modes below 10 Hz, and that response is also generated in the direction transverse to the shaking; the example is shown for L5 y direction. Using the global rational fraction polynomial (GRFP) [21] system identification procedure with FRFs merged between two setups for x direction (shaking and response) and y direction (shaking and response) produces the estimates presented in Table 3.

Modal masses are critical parameters that directly control dynamic response, but they can be misunderstood because of the way they are linked to mode shape scaling. Modal mass is the integral with respect to height of mass weighted by squared horizontal modal ordinate [22]. In the context of predicting response of the masonry structure the most useful scaling method sets the mode shapes to have value 1.0 (unity) at the highest (gallery) level in the masonry tower where the shaker is (usually) located. Then the physical (and modal) response at this location is obtained by considering each mode as a single degree of freedom system with this 'unity scaled' value of modal mass, as given in Table 3.

The circle fit (CFIT) [20,23] single-mode system identification procedure was also applied to check the GRFP estimates, appearing

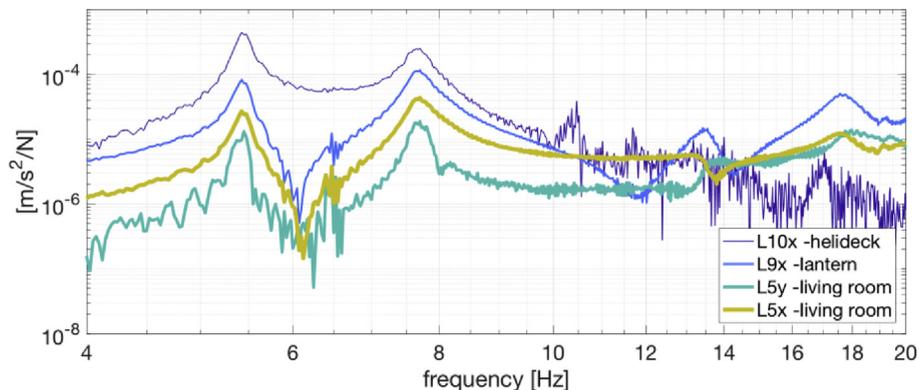


Fig. 7. Les Hanois frequency response functions for run 7 x-direction shaking with legend indicating sensor location and orientation.

**Table 3**  
Les Hanois modal property estimates from circle fit and GRFP.

Direction	Mode	Frequency/Hz		Damping/%		Modal mass/tonnes	
		CFIT	GRFP	CFIT	GRFP	CFIT	GRFP
x	1	5.44	5.45	1.08	0.98	553	639
x	2	7.63	7.76	2.02	2.42	215	229
y	1	5.46	5.45	1.05	0.92	572	606
y	2	7.71	7.77	2.45	2.09	197	221

to show high-quality identification for the fundamental mode with each shaker direction. The circle fit is a classical technique that fits to the 'Nyquist' plot of the real and imaginary components of the frequency response function. The natural frequency occurs where circle sweeps fastest through the data points which are at equal frequency intervals, the damping is available from the angle between data points at this frequency and the modal mass is derived from the radius of the circle. The method provides a powerfully visual but sometimes misleading way to estimate modal parameters. Fig. 8 shows the fit to 'point mobility' acceleration FRF relating response to force at L9 for 7–9 Hz range. Modal parameter estimates using CFIT are also presented in Table 3.

Estimates using CFIT differ from those obtained using GRFP and from those obtained using OMA (NExT/ERA) (Fig. 6). In each case the estimates are reported to two significant figures so that the small differences can be observed, although the precision of such estimates is open to question, with some dependence of estimates on analyst skills and preferences and on data selection. Error bounds on (and hence precision of) parameter estimates are not available for the methods used, but are available for a different class of techniques using Bayes' theorem [24,25].

The modal mass values are significant as they directly control the lighthouse response to the wave loading whose characterisation was a stated major objective of STORMLAMP.

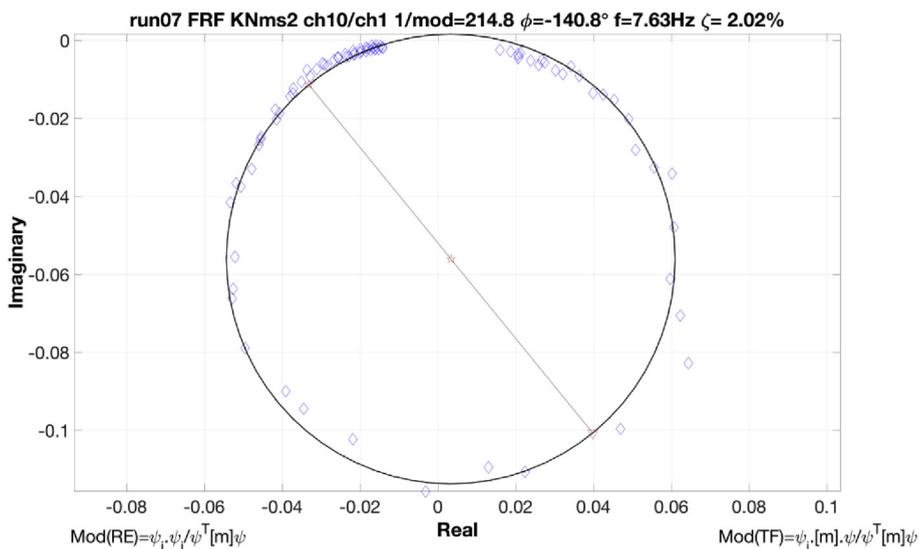
The parameter estimates are slightly different according to the direction of shaking and the alternating sign of the helideck mode shape ordinates between 1st and 2nd modes in each direction indicated in Fig. 6 is clearer from the forced vibration mode shapes. Both directional and helideck effects are discussed later.

Experience at Les Hanois was useful for refining subsequent lighthouse modal tests. Forced vibration testing proved to be the more viable approach and required swept sine signal generation to achieve adequate SNR indicated by coherence functions for point mobility (L9) FRFs in Fig. 7 mostly above 0.95. Subsequent tests used longer sweep durations (but fewer averages) for better frequency resolution and ensuring good SNR.

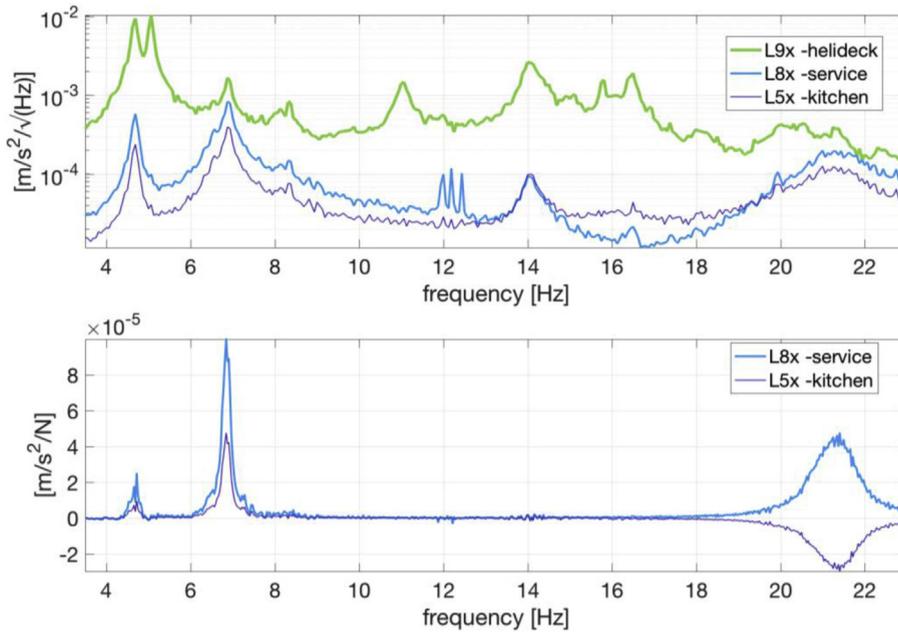
**5. Variations on modal test procedure for other lighthouses**

Measurement techniques depended on site conditions such as time on station and weather conditions, and while patterns emerged in modal parameters, each lighthouse had its own unique modal features.

Wolf Rock has nine levels from entrance (L1) to helideck (L9). L8 is the service room atop the masonry tower, and there is also a mezzanine walkway (not counted as a level) around the steel lantern enclosure. The same approach was used as at Les Hanois, of reference accelerometers in both axes and moving accelerometers between levels. There is no clear recurring internal reference



**Fig. 8.** Circle fit to L9 point mobility acceleration FRF for Les Hanois mode 2.



**Fig. 9.** Wolf Rock response. Upper: Ambient response using logarithmic scale; Lower: imaginary part of FRF for 3–30 Hz swept sine. Legends indicate sensor location and orientation.

feature so x direction was set in the approximate southerly direction and  $45^\circ$  anti-clockwise from the axis of the entrance door, which is set in a 2.3 m deep corridor at the entrance level. Compass bearing for positive x direction was  $168^\circ$ , with y direction at  $78^\circ$ .

Broadband random shaking in the range 3–30 Hz was used for the first setup, but the shaker response was drowned in ambient response, with unusable signal to noise ratio. Swept sine excitation in 3–30 Hz range provided some improvement in SNR but not in the low frequency range, so 3.4–8 Hz sweep was used to prioritise identification of first two modes in four setups (i.e. two sets of accelerometer locations and two shaker directions). As at Les Hanois, time was short, and problems with the shaker limited duration of the four swept sine measurements to less than the desired 900 s.

Fig. 9 shows ambient response spectra (upper plot), with mains subharmonics above 12 Hz, an apparent structural mode at 14 Hz and some helideck-only modes. This 14 Hz mode does not appear in the FRF for 3–30 Hz swept sine excitation in x direction only (lower plot, imaginary part of FRF), whereas the sign switch for the mode above 20 Hz suggests it is the second cantilever mode. The coherence function around the first mode (below 5 Hz) is below 0.5, but 3.4–8 Hz swept sine provides clearer identification of both modes below 8 Hz.

**Longships.** Significant time was spent moving accelerometers between levels at Les Hanois and Wolf Rock to suit the OMA system identification process of gluing mode shape pieces but which yielded poor results. Hence a more efficient arrangement optimised for shaker testing was planned for Longships.

Longships has ten levels including a mezzanine level around the lantern. The shaker was set on L8, the highest point on the masonry tower. Nominal y direction was aligned with the tunnel to the main entrance through the thick tower base, with compass bearings  $155^\circ$  for x and  $65^\circ$  for y.

Despite perfect operation before travel to site, the shaker initially failed to operate but was coaxed to run for x direction only, with 870 s of 2–30 Hz swept sine shaking with 64 s sweep duration. With perfect weather (no strong wind) this was enough to identify x direction modes before the shaker failed completely, but no y direction shaker data are available. Subsequent stripping down of the shaker revealed a bolt had worked loose, and had damaged the armature coil, which was sent for repair. The fault recurred in a subsequent lighthouse test, possibly due to strong vibrations in the helicopter hold.

**Bishop Rock.** The lighthouse has ten levels: Lantern Room (L9) is at the crown of the masonry tower but the shaker could not be set there due to lack of space and was located at L8, with mode shapes unity normalised to L9. Accelerometer compass bearing for x direction was  $350^\circ$  (slightly west of north and aligned with the entrance level corridor to the door) and for y direction  $260^\circ$  (south of west). With the APS 113 HF unserviceable, a lower power APS 113 LF was used, which was not ideal due to Bishop Rock being the largest structure, tested during the windiest conditions. However, even with the poorest coherence functions among the set of modal tests, it was still possible to obtain useful FRFs (Fig. 10) with shaker power focussed in the 3–8 Hz range and sweep duration  $T = 64$  s. For Bishop Rock the FRF (Fig. 10) clearly shows four possible modes for y direction and three for x direction, with only the first two common at the approximate same frequency for both directions and being identifiable in the x direction FRF.

**Dubh Artach** is the only lighthouse of the six without a helideck and access is via a helipad with equipment hoisted by a retractable winch. The lighthouse has seven accessible levels and, unlike the other five structures, the lantern room where the shaker was set has a circular masonry wall above the gallery level. Perfect clear and calm weather meant that shaker testing was expected to

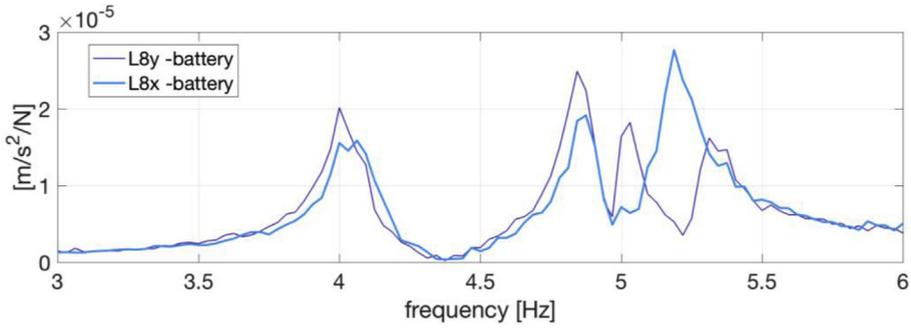


Fig. 10. Bishop Rock point mobility FRFs in y and x directions with legend indicating sensor location and orientation.

provide high quality data, and swept sine excitation with  $T = 64$  s was used to obtain FRFs in the ranges 3–8 Hz and 8–30 Hz. Unlike the other five structures there was only one clear mode in the lower frequency range for each direction (Fig. 11). Dubh Artach has all openings (windows and entrance door) vertically aligned in a single direction which was chosen as the nominal x axis so there is a good chance measurements in the two (x,y) directions would be along principal axes.

**Eddystone** was the last structure to be tested, and longer time on station (with a planned overnight stay) allowed for more extensive measurements. The lighthouse has eleven levels and the shaker was set in the lantern room (L10) atop the masonry tower. With a larger set of 16 accelerometers, only L1-L6 needed accelerometers rotated from x direction (north) to y direction (west) between setups. The entire set of accelerometers was left recording overnight for an extended ambient vibration measurement, which was not possible at other lighthouses. Swept sine excitation in the ranges 3–8 Hz then 6–30 Hz with  $T = 64$  s was used in each direction, allowing clear identification of a set of vibration modes.

### 6. Summary of modal properties

Fig. 12 summarises modal properties for the six lighthouses in the form of mode shapes, frequencies, damping ratios and modal masses for the direction with the lower frequency modes identified using GRFP. The right hand column of Fig. 12 provides numerical values of frequencies, damping ratios and modal masses for the alternate direction. The mode shapes and frequencies in the alternate direction are within a few %, but modal masses can be very different. In all cases the estimates are obtained from GRFP fitting and mode shapes are normalised to unity atop the masonry tower. For Wolf Rock, mode 3 is only partially measured so is interpolated below the gallery, and is set to zero at the helideck where the response coherent with the excitation almost vanishes.

The mode shapes with large helideck ordinate have much larger modal mass. This is because the contribution to the modal mass calculation goes with the square of the modal ordinate. For Eddystone the mass of the horizontal (helipad) part of the helideck is 5.55 t and its modal ordinate (for mode shape scaled to unity at the top of the masonry structure) is 18.89. Scaling the mass by the square modal ordinate indicates a 1980 t contribution to modal mass. Since the open helideck structure is practically transparent to horizontal loads due to breaking wave impacts, response of the masonry towers in these modes would be expected to be relatively low.

### 7. Effects of imperfect axi-symmetry on experimental modal analysis

Any lack of symmetry in lighthouse foundation arrangement and in openings (for windows and entrances) and fittings apparently leads to stiffness that varies by direction. This should result in a pair of vibration modes having very close frequencies and mode shapes that are similar in elevation but orthogonal to each other in plan view. The specific alignment in plan could be estimated by a

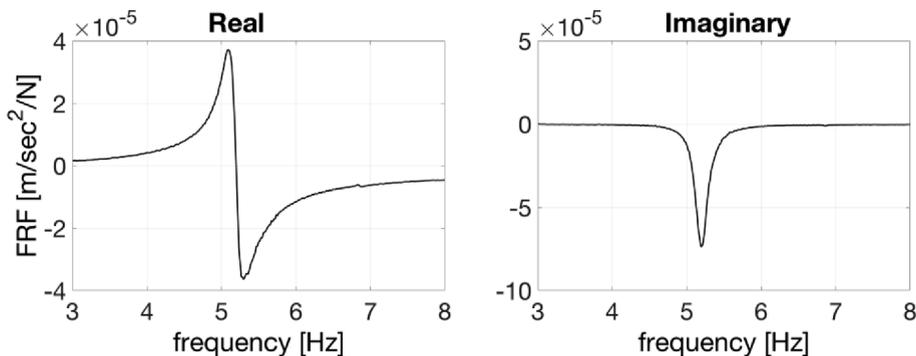


Fig. 11. Real and imaginary parts of FRF for Dubh Artach (y direction).

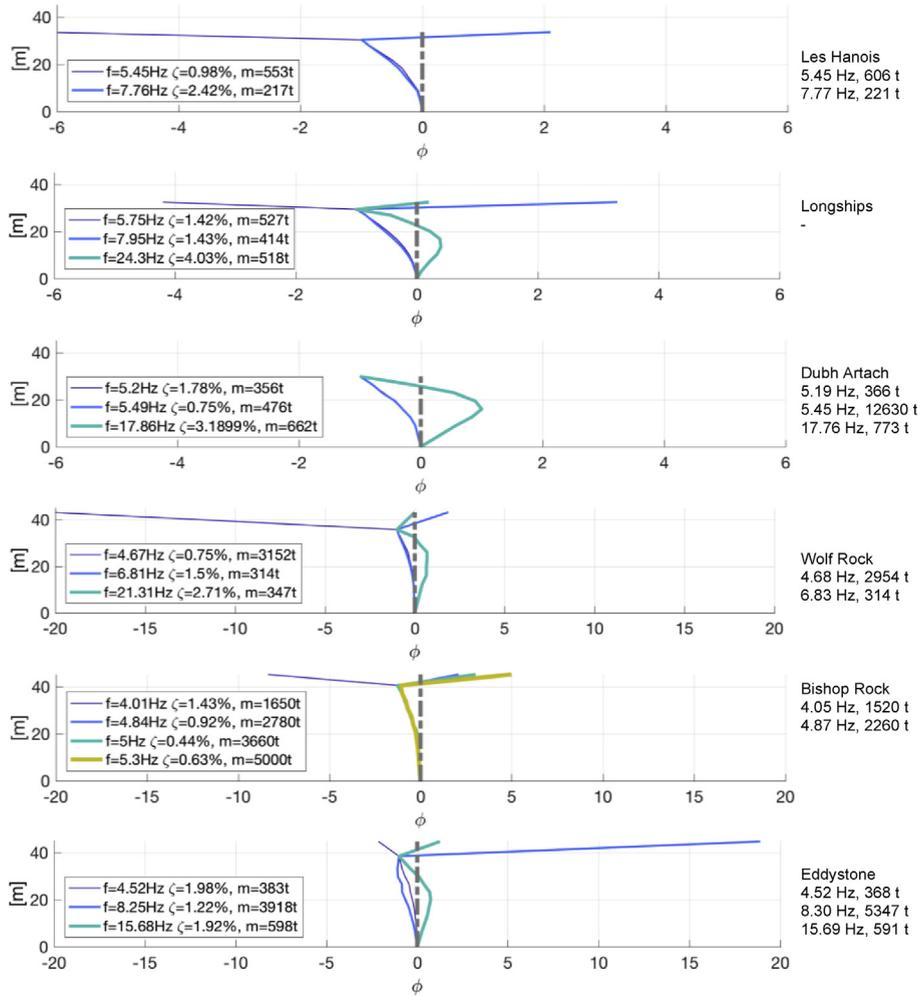


Fig. 12. Modal parameters from GRFP with mode shapes normalised to unity at shaker location. From top: Les Hanois (x), Longships (x), Dubh Artach (x), Wolf Rock (x), Bishop Rock (y), Eddystone (x). Frequency and modal mass estimates obtained from measurements in the alternate orthogonal direction are in right hand column.

finite element model or by modal testing. As demonstrated (Fig. 6), operational modal analysis (OMA) can be used to suggest specific alignment, but success is limited and the clarity of (Fig. 6) was rare among the set of six lighthouses. The result was also obtained well after the test and could not have been obtained immediately while on station. However, the observation from Fig. 7 that FRFs in the direction transverse to the shaking can be non-zero suggests a possible way to study mode alignment from the generally more reliable forced vibration measurements.

Consider the asymmetry of the lighthouse for the simplest mode (zero nodes and cantilever shapes) to be modeled as a lumped mass  $m$  with unequal springs  $k_{x'}$ ,  $k_{y'}$  aligned with local axes that are rotated by an angle  $\theta$  with respect to global axes (Fig. 13)x, y.

The forces and displacements in the local axes relate to those in the global axes via the transformation matrix

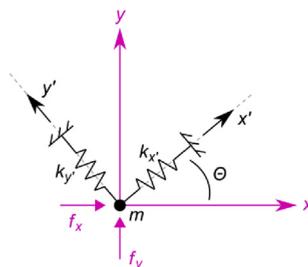


Fig. 13. Rotated axis system.

$$T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \quad (1)$$

where  $T^T = T^{-1}$ .

$$\text{If } v = \begin{bmatrix} x' \\ y' \end{bmatrix}, u = \begin{bmatrix} x \\ y \end{bmatrix}, \text{ then } v = Tu \text{ and } \begin{bmatrix} f_{x'} \\ f_{y'} \end{bmatrix} = T \begin{bmatrix} f_x \\ f_y \end{bmatrix},$$

The mass and stiffness matrices in local coordinates are

$$K' = \begin{bmatrix} k_{x'} & 0 \\ 0 & k_{y'} \end{bmatrix}, K = T^{-1}K'T, M' = M = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \text{ and their eigen-solution is}$$

$$\omega_1 = k_{x'}/m; \omega_2 = k_{y'}/m; [\phi_1 \ \phi_2] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = T^{-1}.$$

Naturally, the mode shapes are along the rotated principal axes (of the different stiffnesses), with normal coordinates being  $v$ , so that equations of motion are

$$M\ddot{v} + K'v = \Phi^T f. \quad (2)$$

Writing  $v(t) = V(\omega)e^{i(\omega t - \phi)}$ ,  $f(t) = F(\omega)e^{i\omega t}$  and introducing (equal damping)  $\zeta$

$$V\ddot{V}(\omega) = M^{-1} \begin{bmatrix} DAF_1 & 0 \\ 0 & DAF_2 \end{bmatrix} \Phi^T F(\omega), \quad (3)$$

where  $DAF_j = \frac{-r_j^2}{1-r_j^2+2i\zeta r_j}$  and  $r_j = \frac{\omega}{\omega_j}$ .

If the force is only in the x direction and dropping the  $(\omega)$ ,

$$\ddot{V} = M^{-1} \begin{bmatrix} DAF_1 & 0 \\ 0 & DAF_2 \end{bmatrix} \Phi^T \begin{bmatrix} F_x \\ 0 \end{bmatrix}, \quad (4)$$

and the response is measured in both x and y directions then

$$\ddot{U} = \begin{bmatrix} \ddot{X} \\ \ddot{Y} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} DAF_1/m & 0 \\ 0 & DAF_2/m \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} F_x \\ 0 \end{bmatrix} e^{i\omega t} \quad (5)$$

i.e.

$$\begin{bmatrix} \ddot{X} \\ \ddot{Y} \end{bmatrix} = (F_x/m) \begin{bmatrix} \cos^2\theta \cdot DAF_1 + \sin^2\theta \cdot DAF_2 \\ \cos\theta\sin\theta \cdot DAF_1 - \sin\theta\cos\theta \cdot DAF_2 \end{bmatrix} e^{i\omega t}. \quad (6)$$

These relationships are simulated via a two-degree of freedom system (using software package NDOF [26]) for  $\zeta = 2\%$  and mass  $m = 200$  t, for stiffnesses  $k$  and  $k + \delta k$  where  $k = 200$  MN/m and  $\delta k$  takes values [0 0.5 1 2.5 5 10 25] %. The same simulations are run for different principal axis rotations  $\theta$  taking values [0 15 30 45]°. Example ‘point mobility’ FRFs for x and y response with respect to forcing in x direction at the same point are given as Nyquist plots in Fig. 14. Note that  $\delta k = 5\%$  corresponds roughly to 2.5% frequency difference.

Fig. 14 shows that the magnitude of the ‘transverse-mobility’ FRF i.e. in the direction transverse to the shaking increases with both stiffness difference (left two plots) and angle (right two plots). Effects of stiffness and hence frequency difference are more obvious in the transverse mobility and of angle in the direct mobility but differentiating the two effects from potentially noisy Nyquist plots alone would be difficult. The best way to detect stiffness/frequency differences would be using an OMA procedure such as SSI or NExT/ERA.

The transverse mobility is visible with different degrees of clarity, among a number of lighthouses. Fig. 15 shows two examples with superposed direct and transverse-mobility FRFs. Eddystone fits the model of significantly rotated principal directions but very small  $\delta k$  because the transverse mobility is not flattened; in fact OMA using the good quality ambient response measurements identifies a pair of modes at 4.45 Hz and 4.54 and rotation of approximately 30°. For Dubh Artach the frequency differences (hence stiffnesses) obtained from OMA of the very weak ambient response are also very small (5.25 Hz/5.38 Hz) so the logical conclusion is that the chosen x and y axes were naturally aligned with the principal axes.

The effect on identification accuracy of having two very close modes can be investigated using the simulated data. GRFP and circle fit were tried on the data of Figs. 14 and 15. Two simulation variants, 15° rotation with 5% stiffness difference referred to as E15\_5, and 45° rotation with 5% stiffness difference (E45\_5) were used. The experimental data from Eddystone for shaking in x direction was also used, and the results are shown in Table 4. The base parameters for the simulations are  $m = 200$  t,  $k = 200$  MN/m<sup>2</sup> and 220 MN/m<sup>2</sup>, with 2% damping, exact mode frequencies 5.0329 Hz and 5.1572 Hz. Modal masses are 214 t and 2787 t for E15\_5, and 400 t (both modes) for E45\_5 with mode shapes normalised to unity in the same DOF (which by symmetry can be either x or y).

Note that GRFP fitting assuming two modes is successful in all three cases for transverse-mobility, whereas GRFP assuming a single mode and circle fit is not. Also the GRFP procedure used cannot be forced to fit two modes to perfect data for unrotated modes.

Circle fit modal mass estimates for direct mobility for 30° rotation are within 5% (of the true value) for stiffness differences up to

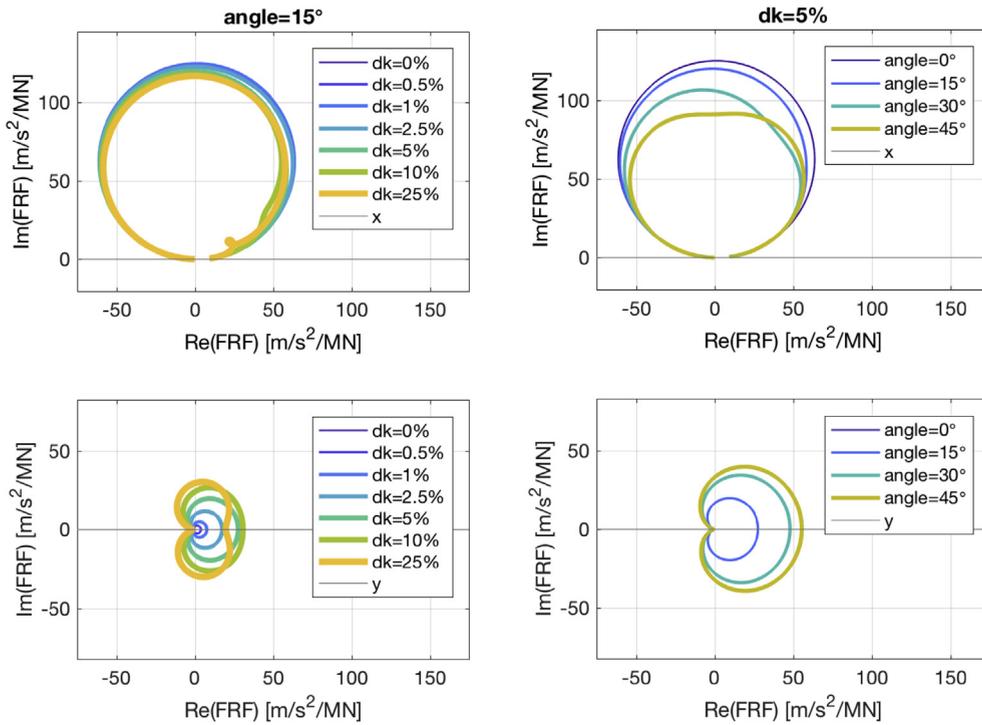


Fig. 14. Nyquist plots for biaxial (x,y) measurement of unidirectional (x) forcing of rotated stiffness system, to same scale. Left:  $\theta = 15^\circ$  with increasing stiffness difference. Right 5% stiffness difference with increasing angle. Upper is the direct point mobility FRF, lower is the transverse mobility FRF.

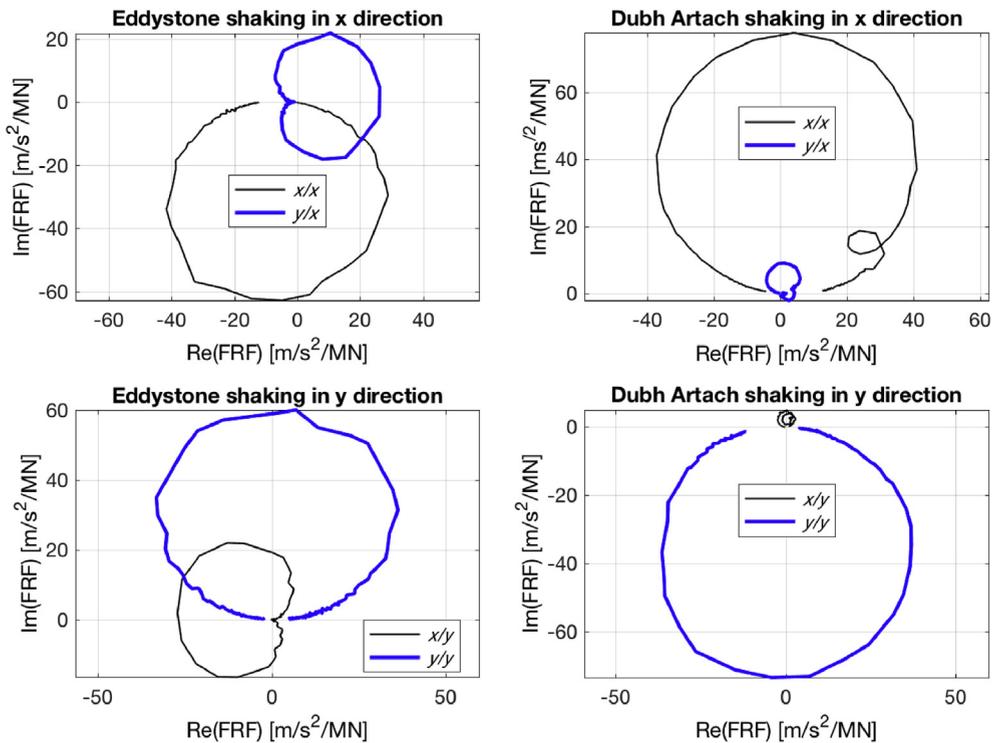


Fig. 15. Nyquist plots of point mobility frequency response functions for Eddystone (left) and Dubh Artach (right). Thin line indicates x direction response, thick line indicates FRF for y direction response.

**Table 4**

Modal identification of experimental data and simulated data for simulated and measured systems with rotated axes using global rational fraction polynomial (GRFP) and circle fit (CFIT).

ID	GRFP 2 modes			GRFP 1 mode			CFIT 1 mode					
	$f_1/\text{Hz}$	$\zeta_1/\%$	$m_1/t$	$f_2/\text{Hz}$	$\zeta_2/\%$	$m_2/t$	$f/\text{Hz}$	$\zeta/\%$	$m/t$	$f/\text{Hz}$	$\zeta/\%$	$m/t$
E15.5 x/x	5.033	2	3000	5.157	2	214	5.152	2.1	198	5.155	2.1	199
E45.5 x/x	5.033	2	400	5.157	2	400	5.095	2.36	197	5.025	3.37	146
E15.5 y/x	5.033	2	800	5.157	2	800	–	–	–	–	–	–
E45.5 y/x	5.033	2	400	5.157	2	400	–	–	–	–	–	–
Eddystone x/x	4.499	1.52	663	4.55	1.41	636	4.529	2	383	4.52	1.99	304
Eddystone y/x	4.484	1.7	765	4.57	1.81	909	–	–	–	–	–	–

5%, whereas for 45° rotation modal mass estimates differ by at least 5% for all but 0% and 1% stiffness difference.

Two main observations can be made:

First, circle fit underestimates modal mass significantly for rotations approaching 45° whereas GRFP for single mode does not, so circle fit should be used with care unless the axis rotation is known to be small. To some extent the likelihood of error due to both rotation and stiffness differences can be judged by comparing the transverse FRF with the direct FRF. GRFP for two close modes clearly works even for FRFs that visually resemble single mode FRFs. Single mode identification should be used when the transverse FRF is negligible.

Second, the single mode modal mass is the inverse of the sum of the inverse masses for the individual modes and the individual mode masses are respectively  $m/\cos^2\theta$  and  $m/\sin^2\theta$ . This is intuitive and consistent with the definition of frequency response function for rotated axes and equation (6).

### 8. Effect of helideck on experimental modal analysis

Fig. 12 shows clearly that for all the helideck equipped lighthouses a pair of zero-node first order modes appears, the lower frequency version having the same sign of tower and helideck modal ordinate, the higher having opposite sign. The same pattern occurs in both directions of shaking and hence in the two principal directions, making a total of four modes with similar mode shape in the tower.

The original focus of the STORMLAMP project was on the behaviour of the masonry towers, which are believed to be potentially more vulnerable to wave loading than the helideck. Hence detailed lighthouse structural simulation using both finite element modelling (FEM) and discrete element modelling (DEM) have so far focused on Fastnet Lighthouse, a masonry-only tower [5] unadorned by helideck. Without a-priori models it was expected that a helideck might act either as a dynamically independent lumped mass or as a short extension to the tower. In the former case the helideck would vibrate almost by itself and in the latter case mode shapes would be continuous with the main structure but with relatively large modal ordinates [27]. In fact, the helideck structures sometimes act in a similar way to a tuned mass damper, with mass only a few % of the main structure but with similar ratios of stiffness to mass as for the masonry tower. This is the most likely situation for Les Hanois and Longships where the helideck mode shape ordinates are similar for the higher and lower frequency modes of a pair.

Fig. 16 shows the structural arrangement of Eddystone Lighthouse helideck as developed elevation of the vertical structure and plan of the helipad (horizontal grillage).

Fig. 17 provides views of the helipad steelwork (infill panels, horizontal members and vertical posts) and the lower part of the vertical structure with beams bolted to the gallery and cruciform sections rising from handrails and posts.

In principle, the modal test results should provide enough information to characterise the essential mass and stiffness properties of the helideck, provided simple assumptions are made about their distribution (i.e. lumping).

Consider the first mode of the masonry structure on its own in either principal direction to be characterised by modal mass  $m$  and modal stiffness  $k$ , and the helideck structure to be characterised by lumped (or modal) mass  $m_h$  and stiffness (lumped or modal)  $k_h$ . These modal parameters are obtained using mode shapes scaled to unit maximum value at the top of the gallery. The undamped equations of motion are then

$$Kx + M \ddot{x} = 0 \quad \text{or} \quad \begin{bmatrix} k_h & -k_h \\ -k_h & k + k_h \end{bmatrix} \begin{bmatrix} x_h \\ x \end{bmatrix} + \begin{bmatrix} m_h & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} \ddot{x}_h \\ \ddot{x} \end{bmatrix} = 0$$

with eigen-solution  $\Omega = \text{diag}\{\omega_1 \ \omega_2\}$ ;  $\Phi = [\phi_1 \ \phi_2]$ .

Mode frequencies, shapes and modal masses  $m_1, m_2$  are all known from the modal test so in principle all four unknown (mass and stiffness) parameters should be identifiable.

#### 8.1. Wolf Rock structural identification

For Wolf Rock Lighthouse,  $\Omega = \text{diag}\{[29.45 \ 42.79]\}$  and modes normalised to unity at the tower (gallery) and corresponding modal masses are

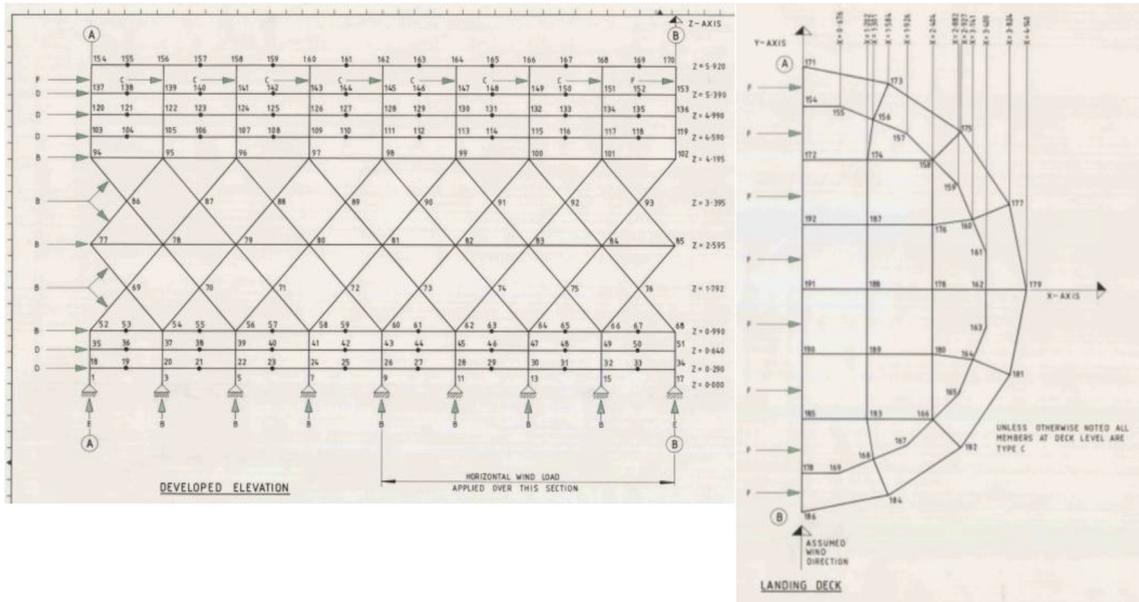


Fig. 16. Eddystone Lighthouse helideck arrangement.



Fig. 17. Eddystone helideck views. Left: helipad, right: vertical structure rising from the gallery, with solar panels attached.

$$\Phi = \begin{bmatrix} 20.22 & -1.86 \\ 1 & 1 \end{bmatrix}, m_1 = 3152 \text{ t } m_2 = 314 \text{ t}.$$

$$\text{Then solving } \Phi^T M \Phi = \begin{bmatrix} 3152 & 0 \\ 0 & 314 \end{bmatrix} \text{ provides } M = \begin{bmatrix} 7.1 & -1 \\ -1 & 285.7 \end{bmatrix}.$$

The estimates  $m = 286 \text{ t}$  and  $m_h = 7.1 \text{ t}$  are reasonable.  $M$  should be diagonal, but the off-diagonal terms are non-zero, probably since the simple two-degree of freedom representation is not totally reliable.

The original Trinity House drawings provide a high level of detail of the courses and can be used to check the tower modal mass estimates. Summing the contributions of the individual courses and taking a density  $2660 \text{ t/m}^3$  (corresponding to Douglass' summary of granite mass and volume) leads to a total  $3337 \text{ t}$  (compared to Douglass'  $3350 \text{ t}$  value). This ignores additional tower mass due to grout, slate flooring, internal fittings, lantern and mechanism etc. Weighting the mass values by square modal ordinate from the modal tests (whose shapes differ slightly between the lower and upper of the pair) results in modal mass estimates for the masonry tower alone of  $204 \text{ t}$  and  $242 \text{ t}$ .

The effective lumped mass of the helideck cannot be known without detail of the mode shape throughout the height, but  $5.6 \text{ t}$  – for the helipad alone – is a minimum and  $11.36 \text{ t}$  – for the entire helideck structure – is a maximum, so the inverse estimate ( $7.1 \text{ t}$ ) is reasonable.

Equivalent stiffness values for masonry tower and helideck can also be estimated using

$$K = M \Phi \Omega^2 \Phi^{-1}.$$

Taking the above estimated lumped mass estimates,

$$K = \begin{bmatrix} 6.765 & -11.7 \\ -12.66 & 507 \end{bmatrix} \text{ MN/m}.$$

The off-diagonal terms are not feasible (they should be the negative of the 1,1 term) and the tower stiffness and mass values would suggest a natural frequency (7.93 Hz) for the bare tower, higher than either of the two frequencies from the tower/helideck system rather than somewhere between them.

Taking the estimated helideck lateral stiffness (6.765 MN/m) with the 11.36 t total mass gives a lower bound estimate of 3.89 Hz natural frequency for the (approximated) helideck single degree of freedom system. Simple simulations attaching this single degree of freedom (SDOF) system to a masonry tower having a much higher first mode natural frequency (e.g. above 7 Hz) shows that for the two resulting modes one has a frequency very close to the helideck SDOF mode, the other has a frequency close to the (bare) tower first mode. The mode shapes have the same character of mode shapes observed experimentally, that is to say the lower frequency mode has a very large ( $> 1$ ) and positive helideck: tower mode shape ratio, while the higher frequency mode has a much smaller ( $\sim 1$ ) but negative ratio.

Of the other lighthouses, mode shapes in Fig. 12 show that only Eddystone exhibits the same effect. For the other helideck equipped lighthouses the pair of mode shapes more closely resemble those of a tuned mass damper; in such a case it is likely that the helideck SDOF frequency is quite close to tower first mode frequency.

The relevance of Wolf Rock is that it was chosen for longer term monitoring (due to its very exposed location) and the limited effect of the helideck simplifies estimation of breaking wave loads since the second ( $\sim 6.8$  Hz) mode behaves almost as if there is no helideck.

## 8.2. Eddystone Lighthouse structural identification

The same structural identification exercise of 8.1 was also attempted on Eddystone because the modal test data are the highest quality and wave-impact response data are available.

For Eddystone Lighthouse,  $\Omega = \text{diag}\{[28.274 \ 51.71]\}$ .

Modes normalised to unity at the tower (gallery) and corresponding modal masses are

$$\Phi = \begin{bmatrix} 2.13 & -18.89 \\ 1 & 1 \end{bmatrix}, m_1 = 383 \text{ t}, m_2 = 3918 \text{ t}.$$

$$\text{Then } \Phi^T \begin{bmatrix} m_h & 0 \\ 0 & m \end{bmatrix} \Phi = \begin{bmatrix} 3918 & 0 \\ 0 & 383 \end{bmatrix}.$$

Solving this (as for Wolf Rock) results in small off-diagonal terms, but values  $m = 349.5$  t and  $m_h = 9.73$  t are reasonable.

As for Wolf Rock, original Trinity House drawings were used to check the reliability of these values starting with mass values for the granite courses. For Eddystone (unlike Wolf Rock) the mode shapes in the tower differ significantly between the lower frequency and higher frequency modes of the pair leading to different modal mass estimates of 291 t and 569 t for the masonry tower alone. Hence the simple assumption of a single value for tower modal mass fails and a more sophisticated analysis would be required. However, the 9.73 t estimate lies between helideck lumped mass values of 5.55 t (total mass at the landing platform level) and 12.28 t (total mass of all steelwork and aluminium panels).

The relative simplicity of Wolf Rock and the slightly lower tower modal mass estimates, along with the more exposed position in the Western Approaches, plus anecdotal evidence from Trinity House crew of discernible tower vibration during severe storms make it the best candidate for monitoring effects of wave loading.

## 9. Wolf Rock monitoring, winter 2017/2018

A single JA-70SA triaxial servo accelerometer was installed in Wolf Rock battery room (L7) on 7th September 2017, with a NI cRIO data acquisition recording data via a NI USB-9234 four-channel digital to analog converter. Communication using 3/4G mobile communication and a high gain antennae was initially established but failed after a few days. During a subsequent visit (6th February 2018) as part of the Trinity House maintenance schedule the USB data storage device was replaced and communications re-established. During the period 9/7/2017 to 6/2/2018 the United Kingdom experienced a number of severe storms including Storms Aileen (12–13/9/2017), Ophelia (formerly Hurricane Ophelia, 15–18/10/2017), Brian (20–22/10/2017) and Eleanor (2–3/1/2018), with acceleration signals occasionally exceeding 20% of gravity (0.2 g,  $\sim 2 \text{ m/s}^2$ ).

Fig. 18 shows 1 h of typical wave-driven acceleration response during the period between Ophelia and Brian. The sample shows strong response in modes 2 and 3 (6.8 Hz and 21.3 Hz) while the low response in mode 1 is a consequence of high modal mass previously noted due to the large helideck modal ordinate.

A simple method for estimating wave loading from response data uses the impulsive nature of the response evident in Fig. 18 and shown as time series (Fig. 19) due to the wave impact occurring at about 365 s.

Since impulse equals change in momentum, the known change in velocity (2.8 mm/s) and modal mass of (444 t, correcting twice for the smaller modal ordinate at L7) provides a modal impulse estimate of  $\sim 1.3$  kNs. This is equivalent to an impulse applied exactly at L7. From the Eddystone study [4] it is known that horizontal wave impacts are focused on the lower part of a lighthouse tower and last less than 1 s. Because the modal impulse is the length (and time) integral of force scaled by modal ordinate the actual forces would be orders of magnitude higher. To indicate likely order of magnitude, a simulation exercise for wave impact on Eddystone Lighthouse [5] used a 4.4 MN peak breaking wave force with duration  $< 0.25$  s.

The velocity transient in Fig. 19 is one example from a period summarised in Fig. 20 that included all the significant storms of winter 2017–2018, those having greatest effect being Ophelia, Brian and Eleanor and delivering much larger modal impulses, up to

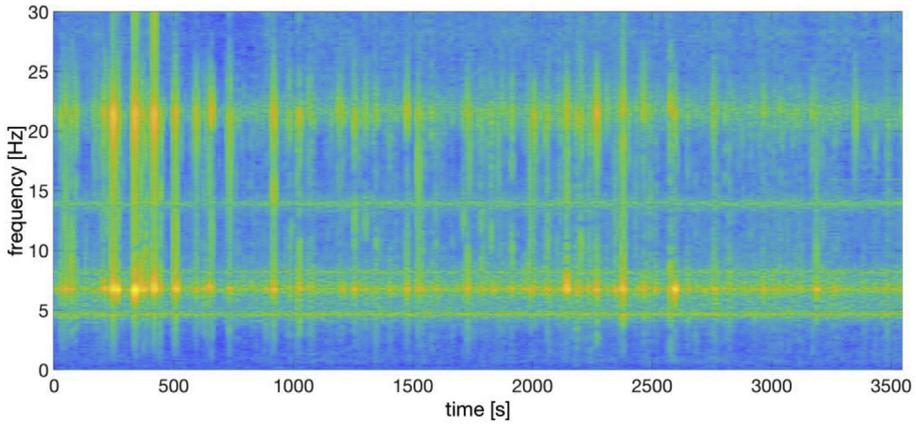


Fig. 18. Spectrogram of Wolf Rock acceleration response 19th October 2017, from 5p.m.

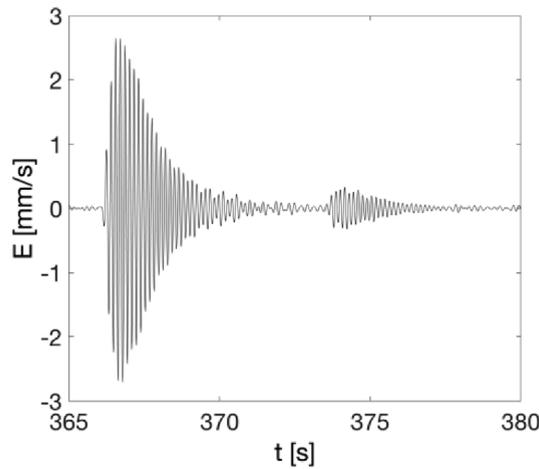


Fig. 19. Wolf Rock velocity response decay from wave impact shortly after 5p.m 19th October 2017.

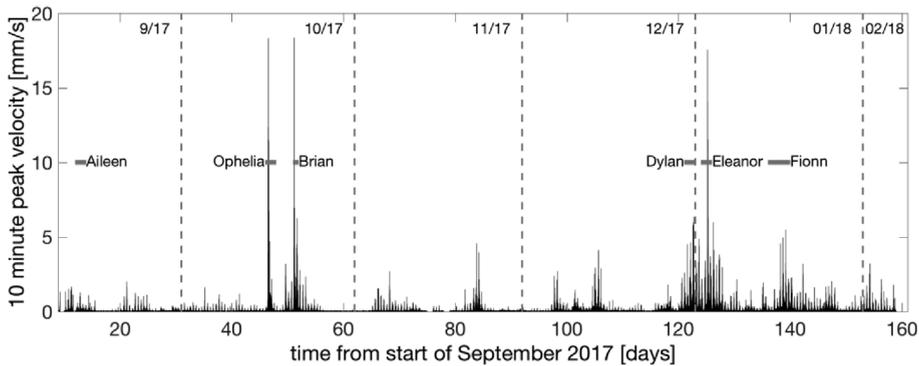
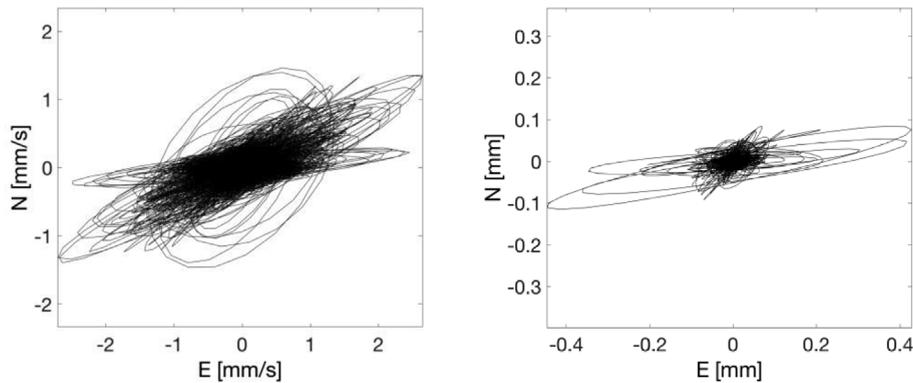


Fig. 20. Velocity maxima during winter 2017–2018.

8 kNs.

The strong storms having greatest effect would have wind-driven waves from west or south west directions in the Western Approaches, and this is borne out by orbit plots, Fig. 21, of Wolf Rock response. The response during Storm Brian was the strongest observed, generating  $\sim 2 \text{ m/s}^2$  peak response equivalent to a ‘very strong’ earthquake according to the Mercalli scale. Even so, this corresponds to a sub-mm displacement indicating that the rocking sensation experienced by lighthouse keepers is the consequence of human sensitivity to acceleration rather than structural distress.



**Fig. 21.** Wolf Rock response to wave impacts (left) as velocity for 19th October 2017 showing alignment with west of south west direction and (right) as displacement for largest response, during storm Brian, 21st October.

## 10. Discussion: challenges and implications

This exercise appears to be the first reported modal testing of any lighthouse structure worldwide, although there have been limited response measurements on structures such as Eddystone Lighthouse [4]. While the modal testing procedure itself is a standard methodology traditionally deployed on a range of structures such as aircraft, footbridges and floors, the interest is in the various challenges and lessons learnt from the experimental programme, the subsequent modal analysis and the insights into the structural behaviour of these iconic structures.

Apart from logistical difficulties (extreme time pressure, access constraints and, strangely, shaker failure due to shaking by helicopter) there were technical challenges in finding the best approach to modal analysis. An approach optimised for OMA aimed to identify horizontal mode ordinates without assumption of directionality but this only worked in a few cases and was too difficult to manage while on station.

The only viable type of shaker (APS 113) turned out to perform surprisingly well, with careful selection of excitation signal, even managing to recover FRF for the most massive structure in the worst weather (Bishop Rock), but the choice of shaker direction depended on lighthouse internal layout and recovery of principal directions was not directly possible. Detailed examination of direct and transverse-mobility FRFs supported OMA identification of close modes.

The behaviour of the helidecks and their effect on modal properties was another curiosity with significance for lighthouse response to breaking wave impact. Apart from having their own dynamics that would require a dedicated experimental investigation, the retrofitted structures appear for many modes to have very large horizontal modal ordinates compared to the masonry tower and can be ‘lively’ during strong winds, even before considering the effects of vertical and horizontal wave loads. In all senses, it seems that Wolf Rock is an excellent candidate for monitoring.

Overall, the modal test results were good enough to provide modal properties that support other parts of the research project such as numerical modelling and wave loading studies.

## 11. Conclusions

Despite the constraints imposed by their remote locations and the environmental loads they experience, the rock-mounted Victorian era lighthouses are ideally suited to modal testing using a small electrodynamic shaker and a limited array of wired high-quality accelerometers. Close examination of the FRFs shows that circle-fitting is not a reliable method for modal parameter estimation due to closeness of modes for many of the lighthouses. Operational modal analysis turns out to be much more challenging mainly due to lack of repeatability and variable directionality.

The lighthouses tested have fundamental (horizontal) modes in the range 4–6 Hz that indicate stiff structures with no strongly anomalous behaviour. Modal masses vary according to mode and structure, but values are as low as 200 t in the case of Wolf Rock for the second mode at 6.81 Hz. Each lighthouse has its own unique dynamic features, and among the structures tested, Bishop Rock has the largest modal masses, consistent with its height and strengthening.

The five helideck-equipped structures show interesting behaviour due to the 20th century retrofit with the lowest cantilever mode in the masonry tower having a pair of modes with alternating phase of helideck motion and, often comparatively very large modal ordinates. Further, the symmetry of the lighthouses results in pairs of modes in more or less orthogonal principal directions that lead to non-zero frequency response functions in directions transverse to shaking.

Based on the outcomes of the set of modal tests, due to its relative simplicity, low modal mass estimates, exposed position in the Western Approaches and anecdotal evidence of tower vibration during storms, Wolf Rock Lighthouse was chosen for long term monitoring. Data from the 2017–2018 winter storm season show clear effects of impulse (breaking) waves, whose effects can be used with the modal test data to characterise the wave loads.

## Acknowledgements

The research has been supported by EPSRC (grant references EP/N022947/1, EP/N022955/1 and EP/NO23285/1) and the UK General Lighthouse Authorities. We are very grateful to Ian Moon, Karen Faulkner and Vincent Ao for their valuable contribution to the field testing.

## Appendix A. Supplementary data

Supplementary data related to this article can be found at <https://doi.org/10.1016/j.marstruc.2018.07.001>.

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